ECE 603 - Probability and Random Processes, Fall 2016

Homework #5

Due: 11/04/16, in class

1. Let X be a Gaussian random variable with mean $\mu = 3$ and variance $\sigma^2 = 1$. Where needed, use the $\text{erf}(\cdot)$ function, as defined in class.

   (a) Roughly sketch $f_X(x)$, the probability density function of X.

   (b) Find the probability that $-4 \leq X \leq 1$.

   (c) Find the probability that $X^2 \geq 10$.

2. You have a table that gives you the value of the "Goeckel Function" for all $x \geq 0$:

$$\mathcal{G}(x) = \int_x^\infty \frac{1}{2} e^{\frac{-u^2}{5}} du$$

   Let Y be a Gaussian random variable with mean μ and variance σ^2; that is, $Y \sim N(\mu, \sigma^2)$. Write an expression for $P(Y \leq y)$ for all y in terms of $\mathcal{G}(x)$.

3. Suppose the discrete random variables X and Y have the probability description:

$$P(X = x, Y = y) = \begin{cases} c|x+y| & x = -2, 0, 2; y = -1, 0, 1. \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the value of the constant c.

   (b) Find the marginal probability description of Y (i.e. $P(Y = y)$ for all y.)

   (c) Find the marginal probability description of X.

   (d) Find $E[XY]$.

   (e) Find $P(Y < X)$.

   (f) Find $P(Y = X)$.

   (g) Sketch $p_{Y|X=-2}(y|X = -2)$, $p_{Y|X=0}(y|X = 0)$ and $p_{Y|X=2}(y|X = 2)$.

   (h) Are X and Y independent?

4. Random variables X and Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} c(x+y), & 0 < y < 1, 0 < x < 1-y. \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find c.

(b) Determine $f_X(x)$ and $f_Y(y)$, the marginal probability density functions of X and Y, respectively. Are X and Y independent?

(c) Find $P(Y > X)$.

(d) Find $f_{Y|X}(y|x)$, the conditional probability of Y given $X = x$. For your limits, put x between constant bounds, and then give the limits on y as a function of x.

(e) Find $E[X]$ and $\text{Var}[X]$.

(f) Find $E[Y]$ and $\text{Var}[Y]$.

(g) Find $\text{cov}(X, Y)$.

(h) Find $E[X + Y]$. *Hint: This part should be very easy.*

(i) Find $\text{Var}[X + Y]$. *Hint: This part should be pretty easy.*

5. Suppose that X and Y are **independent** random variables with probability density functions:

$$f_X(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} 3y^2, & 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find $E[XY]$.

(b) Find $\text{cov}(X, Y)$.

(c) Find $P(X > Y)$.

6. Let X and Y be jointly Gaussian random variables. Let $E[X] = 1$, $E[Y] = 1$, $E[X^2] = 3$, $E[Y^2] = 8$, and $\rho_{X,Y} = \frac{1}{3}$. Define $Z = 2X - 3Y$. Find $P(Z > 3)$.