ECE 564/645 - Digital Communications
Homework #5
Spring, 2014

Due: April 18, 2013

1. [Thank Shamit for this one.] (a) Prove the Singleton bound: for any (n, k) linear block code,

$$d_{\text{min}} \leq n - k + 1.$$

(*Hint: Think about how to get d_{min} by looking at columns of H*).

(b) Codes that satisfy the Singleton Bound with equality are known as "maximum distance separable." Find a code that satisfies the Singleton bound with equality.

2. Prove that one of the following is true for any linear block code: (i) all of the codewords are of even weight or (ii) exactly half of the codewords are of odd weight and half of the codewords are of even weight.

3. Consider a $(3, 1)$ constraint length 3 convolutional code defined by $\underline{g}_1 = 100, \underline{g}_2 = 110, \underline{g}_3 = 111$, where the bits enter the shift register from the left as in class and the shift register connections in $\underline{g}_i$ are defined left to right.

(a) Draw the shift register generator of this code.

(b) Draw the state diagram that can be used to generate this code.

(c) Suppose that the generator starts in the all zeroes state. Four information bits are input, and then two zeroes are input to the generator to flush the shift register back to the all zeroes state. The bits are transmitted across the AWGN channel using binary antipodal modulation with the mapping $0 \rightarrow 1, 1 \rightarrow -1$. Find the maximum likelihood transmitted sequence in the following situations, *showing a new trellis at time instants 3, 4, 5, and 6 in each case*.

- Suppose the decoder is performing hard-decision decoding and receives: 000,110,010,001,001,000
- Suppose the decoder is performing soft-decision decoding and receives: 0.8 0.2 0.9, 0.3 0.8 0.9, -0.9 0.6 -0.9, 1.2 -0.5 -0.5, 1.0 0.9 -0.5, 1.5 0.6 0.8

(d) Find the transfer function $T(x, y)$ and use it to find:

- The free distance of the code.
- The first three terms of a Union Bound to the soft-decision first event error probability $P_f(E)$.
- The Bhattacharyya Bound to the soft-decision first event error probability $P_f(E)$.
- A combination bound to the soft-decision first event error probability $P_f(E)$, obtained by using the first three terms of the Union Bound and then upper bounding the *remaining* terms with the Bhattacharyya Bound. Why is this combination bound commonly employed ?
- The Bhattacharyya Bound to the soft-decision bit error probability.