1. The number of emails that I get in a weekday (Monday through Friday) can be modeled by a Poisson distribution with an average of $\frac{1}{6}$ emails per minute. The number of emails that I receive on weekends (Saturday and Sunday) can be modeled by a Poisson distribution with an average of $\frac{1}{30}$ emails per minute.

(a) What is the probability that I get no emails in an interval of length 4 hours on a Sunday?

(b) A random day is chosen (all days of the week are equally likely to be selected), and a random interval of length one hour is selected on the chosen day. It is observed that I did not receive any emails in that interval. What is the probability that the chosen day is a weekday?

2. The probability density function of a random variable $X$ is given by $f_X(x)$, where:

$$f_X(x) = \begin{cases} cx^2, & -2 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of the constant $c$.

(b) Find the cumulative distribution function $F_X(x) = P(X \leq x)$.

(c) Find the probability that $X^2 \geq 1$.

(d) Find the probability that $X - 1 \geq -\frac{1}{4}$.

(e) Find $E[X]$, the expected value of $X$.

(f) Find $\text{Var}[X]$, the variance of $X$.

(g) Let $Y = X^2$. Find $f_Y(y)$, the probability density function of $Y$.

3. The money (in thousands of dollars) made from investing in stocks “Ystock” and “Zstock” are modeled as the random variables $Y$ and $Z$, respectively. Assume $Y$ and $Z$ are independent with respective probability density functions $f_Y(y)$ and $f_Z(z)$ as shown below:
(a) You want to make as much money as possible, of course. Which stock would you buy?

(b) Suppose you have forgotten your ECE 314, so you decide to flip a coin to decide which stock to buy. What is the probability that you make more than $400?

(c) You decide to rely on some vague idea of the “law of averages”, and thus buy 20 independent stocks, each of which have the probability density function \( f_Y(y) \). What is the probability that you make more than $400 at least half of the time (\( \geq 10 \) times)?

4. A Gaussian random variable \( X \) has mean 2 and variance 4.

   (a) Find \( P(X < 3) \).

   (b) Find \( P(1 < X < 3) \)

   (c) Find \( P(\{X > 4\} | \{X > 3\}) \)

   (d) Let \( Y = X^2 \). Find \( E[Y] \).

5. A continuous random variable is said to have a Rayleigh distribution with parameter \( \sigma \) if its PDF is given by

\[
f_X(x) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2} u(x)
\]

where \( \sigma > 0 \).

   (a) If \( X \sim Rayleigh(\sigma) \), find \( EX \).
   (b) If \( X \sim Rayleigh(\sigma) \), find the CDF of \( X \), \( F_X(x) \).
   (c) If \( X \sim Exponential(1) \) and \( Y = \sqrt{2\sigma^2 X} \), show that \( Y \sim Rayleigh(\sigma) \).