Hw #4 Solutions

1)

(a) $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$

Already in systematic form $[I_2 \vdots P]$, thus

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(b) There are 2^{n-k} syndromes.

Method: To find an $\underline{e}$ in a given coset, choose columns of H s. th. columns sum to syndrome. The appropriate $\underline{e}$ will have a "1" in these columns.

s	coset			
000	00000	01101	10111	11010
001	00001	01100	10110	11011
010	00010	01111	10101	11000
011	00011	01110	10100	11001
100	00100	01001	10011	11110
101	01000	00101	11111	10010
110	00110	01011	10001	11100
111	10000	11101	00111	01010

(c)

$\hat{\underline{b}}$	$\underline{s} = H\hat{\underline{b}}$	coset leader	$\hat{\underline{b}} \oplus$ coset leader
01100	001	00001	01101
11111	101	01000	10111
01101	000	00000	01101
00010	010	00010	00000

(d) The correctible error patterns are the coset leaders 00000, 00001, 00010, 00100, 01000, 10000, 00011, 00110

$$P(E) = 1 - P(c) = 1 - \sum_{\underline{e} \in E_c} p^{wt(\underline{e})} (1-p)^{n - wt(\underline{e})}$$

$$= 1 - (1-p)^5 - 5p(1-p)^4 - 2p^2(1-p)^3$$

Note how for $p < 1/2$, this is using are syndromes most effectively - to correct the most probable error patterns.

2)

(a)

- G is $k \times n$; thus $k=3$, $n=6$ and

$$r = 3/6 = 1/2$$

- $\underline{u}G$ for all $\underline{u} \in \{0,1\}^3$

$$\mathcal{C} = \{0000000, 0010111, 0101110, 0111001, 1001101, 1011010, 1100011, 1110100\}$$

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$$G = [I \mid A^T] \Rightarrow H = [A^T \mid I]$$

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- $2^{n-k} = 16$

(b) $d_{min} = \min_i wt(\underline{0}, \underline{c}_i) = 4$

Yes. If $d_{min}=5$, I would need 7 codewords of wt ≥ 5 that are distance 5 apart: 1111100 ← can't even find 2!

c)

$\underline{s}$	coset leader
0000	0000000
0001	0000001
0010	0000010
0011	0000011
0100	0000100
0101	0000101
0110	0000110
0111	0010000
1000	0001000
1001	0001001
1010	0001010
1011	1000100
1100	0001100
1101	1000000
1110	0100000
1111	0011000

using $H\underline{e}^T = \sum_i \underline{h}_i e_i$

d) max dist = max wt of a coset leader = 2

e) Receive $\underline{y}$

Find $\underline{s} = H\underline{y}^T$. If the coset leader for $\underline{s}$ has wt 0 or 1, correct $\underline{y}$. If the coset leader is wt 2, declare two errors.

3)

Key is to build H! (using $\underline{s} = \sum_i \underline{h}_i e_i$

(b)

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

first 5 cosets

oops!

order reversed

but

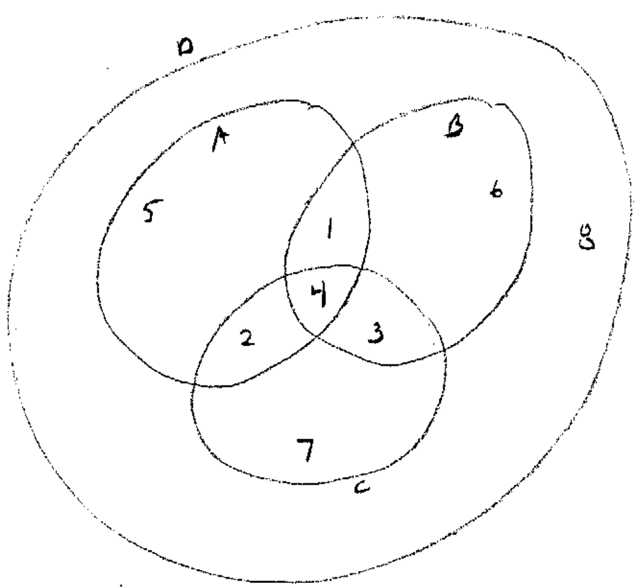
$$\begin{matrix}1\\1\\0\end{matrix} \neq \begin{matrix}1\\0\\1\end{matrix} \oplus \begin{matrix}1\\0\\0\end{matrix}$$

no

(a) yes

$$H = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

4)



(a)

$$H = \begin{array}{c} \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow A \\ \leftarrow B \\ \leftarrow C \\ \leftarrow D \end{array}$$

(b)

$d_{min} = 3$ code <u>extended</u> $\Rightarrow$ $d_{min} = 4$

<u>Algorithm</u>

Look at parity checks A, B, C, D

(1) All satisfied $\Rightarrow$ no errors

(2) Violations in [(A or B or C) <u>and</u> D] or [just D]
- one error - correct as in class

(3) Violation in (A or B or C) <u>and</u> not in D
- declare two errors