ECE 564/645 - Digital Communications, Spring 2014
Homework #4
Due: April 4, 2014 (in class)

1. Consider a (5,2) linear block code for the binary symmetric channel with crossover probability p. Let the code be given by:

$$\mathcal{C} = \{00000, 11010, 01111, 10101\}.$$

(a) Find the generator matrix G and the parity check matrix H for this code.

(b) Give the standard array, denoting the coset leaders.

(c) Use your solution to (b) to decode the following received vectors: 01100, 11111, 01101, 00010.

(d) What are the correctible error patterns ? Using these, find the probability of error of the code as a function of p.

2. A stream of independent information bits, each equally likely to be 0 or 1, are grouped to form row vectors $\underline{u}$ of length 3, which are used by the channel encoder to form codewords by $\underline{u}G$, where

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

<u>Note:</u> Nowhere in this problem do you need to construct the standard array, so do not waste time doing so!

(a) **No justification is required**.

- What is the rate r of this code ?
- Give the set of codewords $\mathcal{C}$.
- Give the parity check matrix H of the code.
- How many syndromes are there?

(b) Does this code have the largest minimum (Hamming) distance of any (7,3) linear block code?

(c) List the syndromes with the coset leaders corresponding to each syndrome.

(d) Consider a binary sequence $\underline{x}$ of length 7 that is chosen so as to maximize the Hamming distance between $\underline{x}$ and the nearest codeword (i.e. $\underline{x}$ is chosen as far away from the code as possible). How far (in Hamming distance) is $\underline{x}$ from the nearest codeword?

(e) Suppose I am transmitting the coded bits over a binary symmetric channel (BSC) with crossover probability $p \ll 0.5$, and at the receiver I want to **simultaneously** correct all error patterns of weight less than two and detect all error patterns of weight two. (In other words, if the error pattern has weight zero or one, I want to output the correct codeword. If the error pattern has weight two, I want to indicate that there is more than one error.)

Give an **algorithm** that performs this error correction/detection. This algorithm should start with the length 7 vector $\underline{\hat{b}}$ that is output from the BSC.

3. (a) Consider the following syndromes with their associated coset leaders:

$\underline{s}$	Coset Leader
000	00000
001	10000
010	01000
011	00011
100	00100
101	00010
110	00001
111	10001

Does a linear block code exist with this syndrome to coset leader mapping? If your answer is "yes", give **any one of the following:** the code's generator matrix, parity check matrix, **or** codewords. If your answer is "no", explain why such a code does not exist?

(b) Consider the following syndromes with their associated coset leaders:

$\underline{s}$	Coset Leader
000	00000
001	00001
010	00010
011	00100
100	01000
101	10000
110	11000
111	10001

Does a linear block code exist with this syndrome to coset leader mapping? If your answer is "yes", give **any one of the following:** the code's generator matrix, parity check matrix, **or** codewords. If your answer is "no", explain why such a code does not exist?

4. Suppose we add another circle to the "circle diagram" used in class to introduce the (7,4) Hamming code. Draw the new circle around the entire diagram and place the position 8 in the new circle (but outside the three original circles). Suppose that we add a fourth parity check that checks that the number of ones inside the new circle is even (i.e. it checks for even parity across all 8 positions).

(a) Find the parity check matrix H.

(b) Find the minimum distance of the code. By finding the indication in the circle diagram of two errors, argue that this code can be used *simultaneously* to correct a single error and detect any pattern of two errors.