In general, there are multiple (often many) ways to solve these problems. I give only one possible method.

(a) 
\[ P(A) = \frac{|A|}{|S|} = \frac{17^{10}}{20^{10}} = \left(\frac{17}{20}\right)^{10} \]

(b) 
\[ P(A) = \frac{|A|}{|S|} = \frac{\binom{17}{3}}{\binom{20}{10}} \quad \text{Smaller: each draw without a defective part makes a defective part more likely on the next draw} \]

(c) 
\[ P(A) = \frac{|A|}{|S|} = \frac{\binom{17}{8} \binom{3}{2}}{\binom{20}{10}} \]

(d) 
\[ P(\text{3rd defective on 10th draw}) = P(\text{3 defective in 9 draws} \cap \text{10th draw is defective}) \]
\[ = P(\text{3rd defective}) P(\text{2 defective in 9 draws}) \]
\[ = \frac{1}{11} \cdot \frac{\binom{17}{3}}{\binom{20}{9}} \]
2) (a) \( P(A) = \frac{|A|}{|S|} = \frac{\binom{13}{2} \binom{39}{3}}{\binom{52}{5}} = 0.274 \)

(b) \( S: 2 \) spades, \( H: 2 \) hearts

\[
P(S \cup H) = P(S) + P(H) - P(S \cap H)
\]

\[
P(S \cup H) = 2 \cdot \frac{\binom{13}{2} \binom{39}{3}}{\binom{52}{5}} - \frac{\binom{13}{2} \binom{13}{2} \binom{26}{1}}{\binom{52}{5}}
\]

\[
= 2 \cdot 0.274 - 0.0608 = 0.480
\]

(c) \( D: 2 \) diamonds

\[
P(S \cup H \cup D) = P(S) + P(H) + P(D) - P(S \cap H) - P(S \cap D) - P(H \cap D) + P(S \cap H \cap D)
\]

\[
P(S \cup H \cup D) = 3 \cdot \frac{\binom{13}{2} \binom{39}{3}}{\binom{52}{5}} - 3 \cdot \frac{\binom{13}{2} \binom{13}{2} \binom{26}{1}}{\binom{52}{5}}
\]

\[
= 3 \cdot 0.274 = 0.822
\]

(d) \( P(H \mid S) = \frac{P(S \cap H)}{P(S)} = \frac{\binom{13}{2} \binom{13}{2} \binom{26}{1}}{\binom{52}{5}} \cdot \frac{\binom{13}{2} \binom{26}{1}}{\binom{39}{3}} \cdot \frac{\binom{13}{2} \binom{26}{1}}{\binom{52}{5}} = 0.22 \)
2) (a) \( P_i: 1^{st} \) part defective law of total probability

\[
P(D_1) = P(D_i, W_j) P(W_j) + P(D_i, W_{j2}) P(W_{j2}) + P(D_i, W_{j3}) P(W_{j3})
\]

\[
= \left( \frac{1}{10} \right) \left( \frac{1}{3} \right) + \left( \frac{1}{4} \right) \left( \frac{1}{3} \right) + \left( \frac{1}{12} \right) \left( \frac{1}{12} \right)
\]

\[
= \frac{1}{60} + \frac{5}{60} + \frac{17}{60} = \frac{17}{60}
\]

(b) \( B: \) exactly 8 defective

\[
P(B) = P(B, W_j) P(W_j) + P(B, W_{j2}) P(W_{j2}) + P(B, W_{j3}) P(W_{j3})
\]

\[
= \left( \frac{1}{6} \right) \left( \frac{1}{3} \right) \left( \frac{30}{30} \right) \left( \frac{10}{10} \right) \left( \frac{1}{3} \right) = 0.00366
\]

\[
l_{B} \leftarrow \left( \frac{1}{3} \right)
\]

(c) \( P(W_1|B) = \frac{P(W_1, B)}{P(B)} \)

\[
P(W_1|B) = \left( \frac{1}{6} \right) \left( \frac{1}{3} \right) \left( \frac{30}{30} \right) \left( \frac{10}{10} \right) = 0.0021
\]

\[
P(W_1 | B) = 1 - P(W_1|B) - P(W_{j2}|B) = 1 - P(W_1, B) \quad \text{(or use)}
\]

\[
= 0.9979
\]

(d) \( P(B|D_i) = \frac{P(D_i, B) P(D_i)}{P(D_i)} = \frac{P(D_i, B) P(D_i)}{P(D_i)} \)

\[
P(B|D_i) = \frac{P(B, D_i, W_j) P(D_i, W_j) P(W_j) + P(B, D_i, W_{j2}) P(D_i, W_{j2}) P(W_{j2}) + P(B, D_i, W_{j3}) P(D_i, W_{j3}) P(W_{j3})}{P(D_i)}
\]

\[
= \left( \frac{1}{6} \right) \left( \frac{1}{3} \right) \left( \frac{30}{30} \right) \left( \frac{1}{3} \right) + \left( \frac{1}{4} \right) \left( \frac{1}{3} \right) \left( \frac{20}{20} \right) \left( \frac{1}{3} \right) = 0.0103
\]

(continued)
Another way:

\[
P(\text{B|10}_3) = \frac{P(\text{B|10}_3) P(\text{B})}{P(\text{B})} = \frac{8/10 \cdot 0.00366}{17/60} = 0.01033
\]

And another:

\[
P(\text{B|10}_3) = \frac{P(\text{B|0|0}_3)}{P(\text{B})} = \frac{P(\text{B|0|0}_3) \cap (\bigcup_{i=1}^{3} \text{W}_i)}{P(\text{B})}
= \frac{P(\text{B|0|0}_3) \cap (\bigcup_{i=1}^{3} \text{W}_i)}{P(\text{B})}
\]

Using the chain rule:

\[
= \frac{P(\text{B|0|0}_3) P(\text{W}_1|\text{B|0|0}_3) + P(\text{B|0|0}_3) P(\text{W}_2|\text{B|0|0}_3) + P(\text{B|0|0}_3) P(\text{W}_3|\text{B|0|0}_3)}{P(\text{B})}
\]

\[
= \frac{P(\text{B|0|0}_3) P(\text{W}_1|\text{B|0|0}_3) + P(\text{B|0|0}_3) P(\text{W}_2|\text{B|0|0}_3) + P(\text{B|0|0}_3) P(\text{W}_3|\text{B|0|0}_3)}{P(\text{B})}
\]
1) (a) \( E_i \) : first bit in error   \( F_i \) : decoder failure

\[
\cdot P(E_i) = P(E_i|B)P(B) + P(E_i|G)P(G)
\]
\[
= 0.4 \cdot 0.2 + 0.025 \cdot 0.8 = 0.10
\]

\[
\cdot P(F) = 1 - P(F^c) \quad N_0: \text{no errors} \quad N_1: \text{one error}
\]
\[
P(F^c) = P(F^c|B)P(B) + P(F^c|G)P(G)
\]
\[
= P(N_0U N_1|B)P(B) + P(N_0U N_1|G)P(G)
\]
\[
= (P(N_0|B) + P(N_1|B))P(B) + (P(N_0|G) + P(N_1|G))P(G)
\]
\[
= \frac{\binom{7}{0}0.4^7 \cdot 0.6^0 + \binom{7}{1}0.4^1 \cdot 0.6^6}{\binom{7}{0}0.4^0 \cdot 0.6^7 + \binom{7}{1}0.4^1 \cdot 0.6^6} \cdot 0.2
\]
\[
+ \frac{\binom{7}{0}0.025^0 \cdot 0.975^7 + \binom{7}{1}0.025^1 \cdot 0.975^6}{\binom{7}{0}0.4^0 \cdot 0.6^7 + \binom{7}{1}0.4^1 \cdot 0.6^6} \cdot 0.8
\]
\[
= 0.158 \cdot 0.2 + 0.842 \cdot 0.8 = 0.821 \Rightarrow P(E) = 0.179
\]

\[
\cdot P(E|F) = \frac{P(E|F^c)P(F^c)}{P(F)} = 1 - P(F^c) = 0.842 - 0.2 = 0.94
\]

\[\text{from above}\]

(b)

\[
\cdot P(E_i) = P(E_i|B)P(B) + P(E_i|G)P(G) = 0.10 \quad \text{(same as (a))}
\]

\[
\cdot P(F) = 1 - P(F^c) = 1 - P(N_0) - P(N_1)
\]
\[
= 1 - \binom{7}{0}0.1^00.9^7 - \binom{7}{1}0.1^10.9^6
\]
\[
= 0.15
\]

(c) The system in (b)! This is why we interleave in wireless comm systems; you can ask me what that means.
Solution:

The receiver needs to get $k$ correct packets. If the receiver can decode it means that the $n^{th}$ packet was received with no error. Also, out of the first $n - 1$ packets, $k - 1$ packets were received with no error. Let $A$ be the event that the sender sends exactly $n$ packets until the receiver can decode the message successfully. Thus since transmissions are independent, we can write $P(A)$ as:

$$P(A) = P(\text{k-1 error-free transmissions out of n-1 transmissions})$$
$$\times P(\text{last transmission was successful})$$

$$= \binom{n - 1}{k - 1} p^{k-1} (1 - p)^{(n-1)-(k-1)p}$$

$$= \binom{n - 1}{k - 1} p^{k} (1 - p)^{n-k}.$$