ECE 563 - Introduction to Comm/SP, Fall 2017
Homework #1 (PARTIAL)

Due: 9/27/17 (in class)

1. Euler's Identities are give by:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

Use Euler's identities to prove the following trigonometric identities:

(a) $\sin(2\theta) = 2\sin\theta\cos\theta$

(b) $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

(c) Use the result from (b) to show that:

$$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$$

(d) $\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

(e) Use the result from (d) to show that:

$$\sin\theta\cos\phi = \frac{1}{2}\sin(\theta+\phi) + \frac{1}{2}\sin(\theta-\phi)$$

And, now that you have derived them, try to remember these important identities - that is the point of this simple problem!

2. The system defined by

$$y(t) = \frac{1}{\sqrt{2T}}\int_{t-T}^{t+T} x(\tau)d\tau$$

is a normalized finite-time integrator. Per class, nearly any linear system is best analyzed as a filter. This problem performs this analysis for the integrator.

(a) Show that this is a linear time-invariant (LTI) system.

(b) Find the system impulse response $h(t)$. (*Hint: This hint sounds completely obvious, but remember that the impulse response is the output when the input is an impulse.*)

(c) Find $H(f)$, the Fourier transform of $h(t)$.

(d) Find $|H(f)|^2$, the magnitude squared of $H(f)$.

(e) Is this a low-pass, band-pass, or high-pass filter? What happens as T becomes larger?

3. Calculating the power of signals will be important throughout the class. Here are some simple examples of particular interest. In each case, start with the definition of the power P_x, but you can use portions of the derivation from earlier parts of the question to answer later parts of the question.

   (a) Find the power P_x in $x(t) = \cos(2\pi f_c t)$, where f_c is some (large) constant. (Yes, we did this in class.)

   (b) Find the power P_x in $x(t) = \cos(2\pi f_c t + \theta)$, where f_c is some (large) constant, and θ is some constant between 0 and 2π.

   (c) Find the power P_x in $x(t) = A\cos(2\pi f_c t + \theta)$, where f_c is some (large) constant, θ is some constant between 0 and 2π, and A is some positive constant.

   (d) Find the power P_x in $x(t) = Am(t)\cos(2\pi f_c t + \theta)$, where f_c is some (large) constant, θ is some constant between 0 and 2π, A is some positive constant, and $m(t)$ is a signal with bandwidth W much less than f_c (i.e. $W \ll f_c$) and with power P_m.

4. Let's study an important topic by combining the results of the previous two problems together. Suppose I have an integrator from Problem #2 with integration time $T = 0.001$ seconds. Find the power in the signal at the output of that integrator for each of the following inputs:

   (a) $x(t) = 25\cos(2\pi 5000 t)$

   (b) $x(t) = 25\cos(2\pi 50 t + \frac{\pi}{2})$

   (c) $x(t) = 25\cos(2\pi 1000 t)$

5. Understanding the nuances of linear and nonlinear systems is critical to systems engineering. For our first foray into this, let's consider bandwidth expansion.

   (a) Can an LTI system produce the output $y(t) = \text{sinc}^2(t)$ when the input is $x(t) = \text{sinc}(t)$? Be sure to justify your answer.

   (b) A common amplifier model is $y(t) = a_1 x(t) + a_3 x^3(t)$, where a_1 and a_3 are constants that depend on the amplifier.

   - Write the signal Fourier transform of $y(t)$ in terms of the Fourier transform of $x(t)$.
   - Suppose $x(t)$ is a real lowpass signal with Fourier transform $X(f)$ that is non-zero for $|f| < W$ and zero otherwise. What is the maximum bandwidth of $y(t)$? (*Note: A justification with a few words and a good picture is fine here.*)