Final Exam Solutions

ECE 564/645 Spring 2013

1) (a) No. Huffman codes have minimum rate of all lossless codes. But I can find a lower rate code. Example: Assign most probable pair 00000, and give all others length 6 sequences that do not start with 00000 (there are 62 such sequences). For my code $R < 3$.

(b)

input	prob	code
00	0.64	0
01	0.16	10
10	0.16	110
11	0.04	111

$$R = \frac{0.64 + 0.16\times 2 + 0.16\times 3 + 0.04\times 3}{2} = 0.78 \text{ bits/symbol}$$

(c) (X_n, X_{n-1})

input	prob	code
00	$0.9\cdot 0.8 = 0.72$	0
01	$0.4\cdot 0.2 = 0.08$	100
10	$0.1\cdot 0.8 = 0.08$	101
11	$0.6\cdot 0.2 = 0.12$	11

$$R = \frac{0.72 + 0.08\times 3 + 0.08\times 3 + 0.12\times 2}{2} = 0.72 \text{ bits/symbol}$$

2)

$$\int_0^{T_s} s_m(t)\, s_n(t)\, dt$$

$$= \int_0^{T_s} \sum_{j=1}^{N} s_{m,j}\, \phi_j(t) \sum_{k=1}^{N} s_{n,k}\, \phi_k(t)\, dt$$

$$= \sum_{j=1}^{N} \sum_{k=1}^{N} s_{m,j}\, s_{n,k} \underbrace{\int_0^{T_s} \phi_j(t)\, \phi_k(t)\, dt}_{\delta_{j,k}}$$

$$= \sum_{j=1}^{N} s_{m,j}\, s_{n,j}$$

3) (a) $s_1(t) = \phi_1(t)$ $(1,0)$

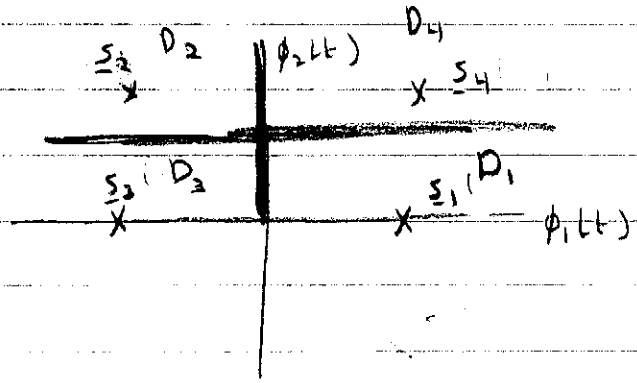
$$s_2(t) = \sqrt{\frac{4}{T_s}}\left(\cos(2\pi f_c t)\cos(\pi/4) - \sin(2\pi f_c t)\sin(\pi/4)\right)$$

$$= \phi_2(t) - \phi_1(t) \qquad (-1, 1)$$

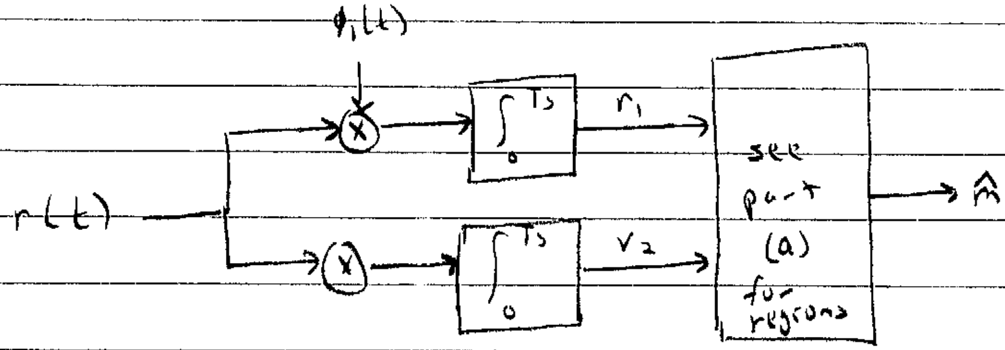
$$s_3(t) = -\phi_1(t) \qquad (-1, 0)$$

$$s_4(t) = \sqrt{\frac{4}{T_s}}\left(\cos(2\pi f_c t)\cos(-\pi/4) - \sin(2\pi f_c t)\sin(-\pi/4)\right)$$

$$= \phi_2(t) + \phi_1(t) \qquad (1, 1)$$



(b)



(c)

$$P(E) = P(E \mid s_2) \qquad \text{(by symmetry)}$$

$$= P(\{n_1 \geq 1\} \cup \{n_2 \leq -1/2\})$$

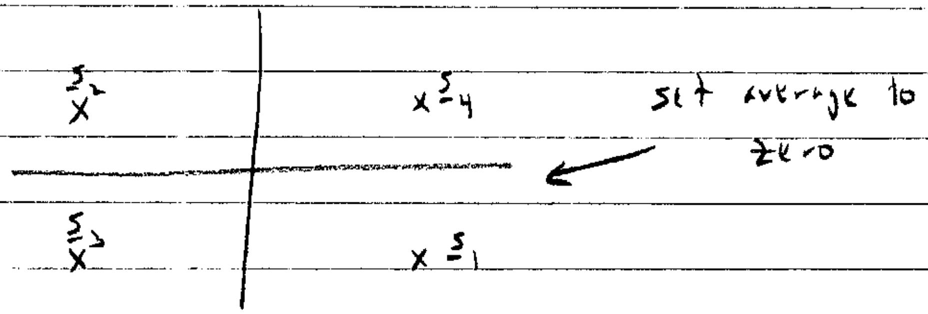
independent

$$= 1 - P(\{n_1 \leq 1\} \cap \{n_2 \geq -1/2\})$$

$$= 1 - \left(\left(1 - Q\left(\frac{2}{\sqrt{2N_0}}\right)\right) \cdot \left(1 - Q\left(\frac{1}{\sqrt{2N_0}}\right)\right)\right)$$

$$= Q\left(\frac{2}{\sqrt{2N_0}}\right) + Q\left(\frac{1}{\sqrt{2N_0}}\right) - Q\left(\frac{2}{\sqrt{2N_0}}\right) Q\left(\frac{1}{\sqrt{2N_0}}\right)$$

(d)



avg. energy goes from $\dfrac{1^2 + (1^2+1^2) + 1^2 + (1^2+1^2)}{4} = 3/2$

to $1^2 + (1/2)^2 = 5/4$ gain is $10 \log_{10} \dfrac{3/2}{5/4} =$

$10 \log_{10} 6/5$

4)

(a)

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$d_{min} = 3$ (all column distinct, so $d_{min} \geq 3$, but $\underline{h}_1 \oplus \underline{h}_4 \oplus \underline{h}_6 = 0$, so $d_{min} \leq 3$)

syndrome	leader
000	000000
001	000001
010	000010
011	001000
100	000100
101	100000
110	010000
111	010001 ← there are other possible answers

(b) Yes. Suppose $d_{min} = 4$. Then, there must exist 7 codewords of wt ≥ 4 that are all distance 4 apart.

111100
001111
110011 ← can't find a fourth!

(c)

Need Euclidean distances:

all 0's: +3 +3 +3

other codewords				$d_E^2(c_0, c_i)$
100101	-3	+1	+1	44
010110	+1	+1	-3	44
001011	+3	-3	-1	52
110011	-1	+3	-1	32
101110	-3	-1	-3	88
011101	+1	-1	+1	24
111000	-1	-3	+3	52

$$P(\epsilon \mid c_0) = Q\left(\sqrt{\frac{24}{2N_0}}\right) + Q\left(\sqrt{\frac{32}{2N_0}}\right) + 2Q\left(\sqrt{\frac{44}{2N_0}}\right)$$

$$+ 2Q\left(\sqrt{\frac{52}{2N_0}}\right) + Q\left(\sqrt{\frac{88}{2N_0}}\right)$$