ECE 697U - Error Control Coding
Homework #1
Spring, 2008

Due: March 5, 2008

1. (a) Argue that for any binary strings $\underline{x}, \underline{y}, \underline{z}$ of length n that

$$d_H(\underline{x}, \underline{y}) \le d_H(\underline{x}, \underline{z}) + d_H(\underline{z}, \underline{y}).$$

(b) Use your result from (a) to show that a code correct any set of t errors as long as:

$$t \le \lfloor \frac{d_{min}(\mathcal{C} - 1}{2} \rfloor$$

2. Consider a (5,2) linear block code for the binary symmetric channel with crossover probability p. Let the code be given by:

$$\mathcal{C} = \{00000, 11010, 01111, 10101\}.$$

(a) Find the generator matrix G and the parity check matrix H for this code.

(b) Give the standard array, denoting the coset leaders.

(c) Use your solution to (b) to decode the following received vectors: 01100, 11111, 01101, 00010.

(d) What are the correctible error patterns ? Using these, find the probability of error of the code as a function of p.

3. Suppose we add another circle to the "circle diagram" used in class to introduce the (7,4) Hamming code. Draw the new circle around the entire diagram and place the position 8 in the new circle (but outside the three original circles). Suppose that we add a fourth parity check that checks that the number of ones inside the new circle is even (i.e. it checks for even parity across all 8 positions).

(a) Find the parity check matrix H.

(b) Find the minimum distance of the code. By finding the indication in the circle diagram of two errors, argue that this code can be used *simultaneously* to correct a single error and detect any pattern of two errors.

4. A customer has a (binary) channel that accepts strings of length $n = 7$, and the only error patterns that *ever* occur are:

$$0000000, 1000000, 1100000, 1110000,$$
$$1111000, 1111100, 1111110, 1111111.$$

Design a linear block code with as high a rate as possible that will correct *all* such error patterns.

5. (a) Find the coset weight enumerators for the (7,4) Hamming code. (You should do this without writing out all of the cosets).

   (b) Suppose that we *extend* the (7,4) Hamming code to get an (8,4) extended Hamming code. Find the coset weight enumerators for this code. (Once again, you should do this without writing out all of the cosets).

6. (a) We construct a code $\mathcal{C}_1$ as follows: take a cube, and label the vertices with bits so that at each vertex, the number of ones at that vertex and the three adjacent vertexes is even. Is this a linear code? Find the blocklength, rate, and minimum distance.

   (b) We construct a code $\mathcal{C}_2$ as follows: take a cube, and label the vertices such that the number of ones on each face is even. Is this a linear code? Find the blocklength, rate, and minimum distance. How does it relate to $\mathcal{C}_1$?

7. How many distinct (7,4) Hamming codes are there? (Note that two codes are distinct if and only if they have a different set of codewords. All (7,4) Hamming codes are equivalent to each other, but they do not have the same codewords).