ECE 603 Fall, 2016

1) 17 good, 3 defective

(a) First 10 are good

$$\frac{\binom{17}{10}}{\binom{20}{10}}$$

(or

$$\underbrace{\frac{17}{20}}_{1^{st}\text{ good}} \cdot \underbrace{\frac{16}{19}}_{2^{nd}\text{ good} \mid 1^{st}\text{ good}} \cdot \underbrace{\frac{15}{18}}_{3^{rd}\text{ good} \mid 1^{st},\, 2^{nd}\text{ good}} \cdots \frac{8}{11}$$)

(b)

$$\frac{\binom{17}{8}\binom{3}{2}}{\binom{20}{10}}$$

(c)

P(3rd defective observed on 10th draw)

$$= \underbrace{P(3^{rd}\text{ defective on }10^{th} \mid 2\text{ defective in }1^{st}\ 9)}_{1/11}\ \underbrace{P(2\text{ defective in }1^{st}\ 9)}_{\frac{\binom{17}{7}\binom{3}{2}}{\binom{20}{9}}}$$

$$= 1/11 \cdot \frac{\binom{17}{7}\binom{3}{2}}{\binom{20}{9}}$$

2) Define:

A: 3 defective out of 5

T_i: shipment from truck i

(a) $P(A) = P(A|T_1)P(T_1) + P(A|T_2)P(T_2) + P(A|T_3)P(T_3)$

$$= \frac{\binom{5}{3}\binom{15}{2}}{\binom{20}{5}} \cdot 1/3 + \frac{\binom{10}{3}\binom{30}{2}}{\binom{40}{5}} \cdot 1/3 + 0 \cdot 1/3$$

(b) $P(T_2|A) = \dfrac{P(A|T_2)P(T_2)}{P(A)}$

$$= \frac{\frac{\binom{10}{3}\binom{30}{2}}{\binom{40}{5}} \cdot 1/3}{P(A)}$$

3) Bernoulli Trials!

(a) $p = 0.15 \quad n = 12 \quad k = 4$

$$\binom{12}{4}(0.15)^4(0.85)^8$$

(b) $p = 0.15 \cdot 0.8 = 0.12 \quad n = 12 \quad k = 3$

$$\binom{12}{3}(0.12)^3(0.88)^9$$

(c) Translate to:

P(There are 4 errors <u>and</u> 2 are major, 2 are minor)

$= P(2 \text{ minor and } 2 \text{ major} \mid 4 \text{ errors}) \, P(4 \text{ errors})$

$$= \binom{4}{2}(0.2)^2(0.8)^2 \cdot \binom{12}{4}(0.15)^4(0.85)^8$$

(d)

P(≥ 6 error free trials in n trials)

$$= \sum_{k=6}^{n} \binom{n}{k}(0.85)^k(0.15)^{n-k}$$

<u>Evaluate for different n</u>

n	P(≥ 6 error free trials in n)
6	0.377
7	0.717
8	0.894
9	0.966

<u>n = 9</u>

4)

(a)

probability of success on a given attempt $= p^N$

$P(\text{success on} \leq m \text{ attempts})$

$= 1 - P(\text{failure on first } m \text{ attempts})$

$= 1 - (1-p^N)^m$

(b)

$P(\text{success on} \leq m \text{ attempts})$

$= \left(P(i^{th} \text{ node succeeds in} \leq m \text{ attempts})\right)^N$

$= \left(1 - P(i^{th} \text{ node fails for } m \text{ attempts})\right)^N$

$= \left(1 - (1-p)^m\right)^N$