ECE 603 - Probability and Random Processes, Fall 2016

Homework #3

Due: 10/10/16, in class

1. In a box there are 20 parts, 3 of which are defective. You draw parts from the box one after another without replacement and test them to find the defective ones. Find the probability of the following:

   (a) You have drawn and tested 10 parts and you have not observed any defective parts.

   (b) Within the 10 parts drawn/tested, you have observed 2 defective parts.

   (c) On the 10th draw/test, you observe the third defective part.

2. Suppose that three trucks leave a warehouse with 20 parts in the first truck, 40 parts in the second truck, and 40 parts in the third truck. Suppose that there are 5, 10, and 2 defective parts out of those in the respective trucks. Suppose I receive my shipment of five parts randomly drawn from those on **one** of the trucks, where the three trucks are equally likely to have made my delivery.

   (a) What is the probability that there are exactly three defective parts out of the five parts in my shipment?

   (b) Given that there are exactly three defective parts in my shipment, what is the probability that it came off of the second truck.

3. In each trial of an experiment, an error occurs with probability 0.15. An error is a *major* error with probability 0.2 and a *minor* error with probability 0.8. The experiment is repeated 12 times, and trials are independent of one another.

   (a) What is the probability that an error occurs in exactly 4 of the trials?

   (b) What is the probability that exactly 3 minor errors occur?

   (c) What is the probability that exactly 2 minor and 2 major errors occur?

   (d) What is the number of times the experiment has to be repeated to have 6 or more error-free trials with probability greater than or equal to 0.9?

4. In a particular computer network, the host computer broadcasts a packet of data to N receivers. The host computer then waits to receive an acknowledgment message from each of the N receivers before proceeding to broadcast the next packet. If the host does not receive all acknowledgments within a certain time period, it will rebroadcast the packet. The host computer is then in "retransmission mode". It will continue retransmitting the packet until, for one of the transmissions, all N acknowledgments are received. Then it will proceed to broadcast the next packet.

   Let p be the probability that a given receiver successfully receives a packet from the transmitter and successfully acknowledges that packet reception. Assume that the events of packet transmission/acknowledgment success are independent between receivers or for separate transmission attempts.

(a) For the protocol described above, where we require that all N of the acknowledgments be received in response to a given transmission attempt, find (for arbitrary m) the probability that a packet is successfully broadcast in m or fewer attempts.

(b) Suppose that we change the protocol such that the transmitter only needs to receive acknowledgments from those receivers for which it has not had a positive acknowledgment for this packet on a prior transmission. (For example, suppose there are three receivers: A, B, and C. If receiver A gets the packet on the first transmission and sucessfully acknowledges such, but B and C do not, the transmitter will retransmit. On this second transmission, the transmitter only looks for successful acknowledgments from B and C to declare successful broadcast of this packet.) Find (for arbitrary m) the probability that a packet is successfully broadcast in m or fewer attempts.