ECE 603 - Probability and Random Processes, Fall 2016

Homework #8

Due: December 14, 2016 (in class)

1. Let $X(t)$ be a wide-sense stationary Gaussian random process with mean zero and autocorrelation $R_X(\tau) = e^{-\frac{|\tau|}{4}}$. Let $N(t)$ be a white Gaussian noise process with power spectral density $\frac{N_0}{2}$.

   (a) Find P_x, the power in $X(t)$.

   (b) Is $X(t)$ strict-sense stationary?

   (c) Find $P(X(3) > 2)$.

   (d) Find the power spectral density $S_X(f)$ of $X(t)$.

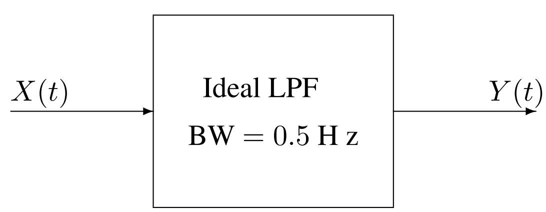
   (e) Find a filter (give $h(t)$ or $H(f)$) that has input $N(t)$ and output with power spectral denisty $S_X(f)$.

   (f) Let $Z = X(0) + X(1) + X(2)$. Find $f_Z(z)$, the pdf of Z.

   (g) Find $P(X(0) + X(1) > 3)$.

2. Suppose that $X(t)$ is a zero-mean Gaussian random process with autocorrelation function $R_X(\tau) = \text{sinc}^2(\tau)$.

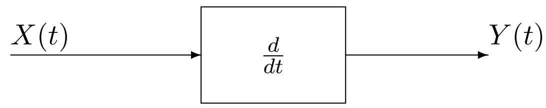
   (a) Suppose that I run $X(t)$ through an ideal lowpass filter with unity gain and bandwidth 0.5 Hz (that is, $H(f)$ = 1 for $|f| < 0.5$ and zero otherwise) as shown below:



   Find:

   - $S_Y(f)$, the power spectral density of $Y(t)$.
   - P_Y, the power in $Y(t)$.
   - $P(Y(0) \geq 2)$.

   (b) Suppose that I run $X(t)$ through an ideal differentiator as shown below:



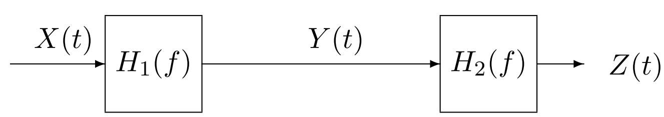
Find:

- $S_Y(f)$, the power spectral density of $Y(t)$.
- P_Y, the power in $Y(t)$.
- $P(Y(0) \geq 2)$.

3. Let $X(t)$ be a wide-sense stationary Gaussian random process with mean $m_X(t) = 0$ and autocorrelation function $R_X(\tau) = 10\delta(\tau)$, where $\delta(\cdot)$ is the Dirac delta function. The random process $X(t)$ is run through a filter combination as shown below, where the first filter has frequency response:

$$H_1(f) = \begin{cases} 1, & |f| < 3 \\ 0, & \text{otherwise} \end{cases}$$

and the second filter has frequency response $H_2(f) = e^{-2f^2}$.



(a) Find the power $E[Y^2(t)]$ in $Y(t)$.

(b) Find the power $E[Z^2(t)]$ in $Z(t)$. **Be sure to simplify your answer as much as possible.**

4. Let $X(t)$ be a stationary Gaussian random process with mean zero and autocorrelation function $R_X(\tau) = 20\text{sinc}^2(80\tau)$. Let $Y(t) = 4\cos(2\pi 30t) + X(t)$.

(a) Find the power $E[X^2(t)]$ in $X(t)$.

(b) For $Y(t)$:

- Find the mean function $m_Y(t) = E[Y(t)]$.
- Find the autocorrelation function $R_Y(t_1, t_2) = E[Y(t_1)Y(t_2)]$.
- Is $Y(t)$ wide-sense stationary?

(c) Is $Y(t)$ a Gaussian random process?

(d) Suppose that $Y(t)$ is run through a filter with impulse response $h(t) = 40\text{sinc}(80t)$ to yield an output $Z(t)$. Define the average power at the output of the filter as:

$$P_Z = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} E[Z^2(t)]dt$$

Find P_Z.