

Homework #1 Solutions

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ECE 564/645

Spring, 2014

1) (a)

$$H(X) = -0.5 \log_2 0.5 - 0.3 \log_2 0.3 - 0.2 \log_2 0.2$$

$$= +1.485$$

		<u>S-F</u>		
(b)		<u>Huff</u>	<u>l(x)</u>	<u>code</u>
AA	0.25	00	2	00
AB	0.15	100	3	100
Ac	0.10	101	4	1010
BA	0.15	010	3	010
BB	0.09	0110	4	0110
BC	0.06	0111	5	01110
CA	0.10	110	4	1100
CB	0.06	1110	5	11100
CC	0.04	1111	5	11110

$$R = \frac{2(0.25) + 3(0.15) + 3(0.10) + 3(0.15) + 4(0.09) + 4(0.06) + 3(0.10) + 4(0.06) + 4(0.04)}{2}$$

$$= \frac{3.0}{2} = 1.5 \text{ bits/symbol}$$

$$H(X) \leq R_{\text{Huff}} \leq H(X) + 1/N$$

$$1.485 \leq 1.5 \leq 1.495 \quad \checkmark$$

(c) (see code above)

$$R = \frac{2(0.25) + 3(0.15) + 4(0.10) + 3(0.15) + 4(0.09) + 5(0.06) + 4(0.10) + 5(0.06) + 5(0.04)}{2}$$

$$= 1.68 \text{ bits/symbol}$$

$$H(X) \leq R_{\text{Huff}} \leq R_{S-F} \leq H(X) + 1/2$$

$$1.485 \leq 1.5 \leq 1.68 \leq 1.985$$

(d) $H(X) \leq R_{\text{Huff}} \leq H(X) + 1/2$

$$1.485 \leq R_{\text{Huff}} \leq 1.61$$

2)(a). The code does not satisfy the Kraft Inequality:

$$1/4 + 5 \cdot 1/8 + 4 \cdot 1/16 = 20/16 = 5/4 > 1!$$

not uniquely decodable

note that 11 is a prefix of 111

- If I change the second codeword from 1001 to 00, I still have a prefix-free code, but now of lower rate; thus, the original code was not a Huffman code.

(b) No.

I can build an $N=4$ code by using the $N=2$ Huffman code twice on each block. Now, I have an $N=4$ code with the same rate as the $N=2$ code. An $N=4$ Huffman code would have a rate no larger than this code.