ECE 313 Fall, 2012

1) (a)

$$x(t) = 3\,\mathrm{rect}\left(\frac{t - 15/2}{5}\right) - 3\,\mathrm{rect}\left(\frac{t + 5/2}{5}\right)$$

(b)

$$X(f) = 3 \cdot 5 \,\mathrm{sinc}(5f)\, e^{-j2\pi \cdot 15/2 f} - 3 \cdot 5\, \mathrm{sinc}(5f)\, e^{+j2\pi \cdot 5/2 f}$$

$$= 15\, \mathrm{sinc}(5f) \left(e^{-j2\pi 15/2 f} - e^{+j2\pi 5/2 f}\right)$$

(c)

$$|X(f)|^2 = X(f) X^*(f)$$

$$= 225\, \mathrm{sinc}^2(5f) \left(e^{-j2\pi 15/2 f} - e^{j2\pi 5/2 f}\right)\left(e^{+j2\pi 15/2 f} - e^{-j2\pi 5/2 f}\right)$$

$$= 225\, \mathrm{sinc}^2(5f)\left(1 + 1 - e^{j2\pi 10f} - e^{-j2\pi 10f}\right)$$

$$= 225\, \mathrm{sinc}^2(5f)\left(2 - 2\cos(2\pi 10f)\right)$$

$$= 450\, \mathrm{sinc}^2(5f)\left(1 - \cos(2\pi 10f)\right)$$

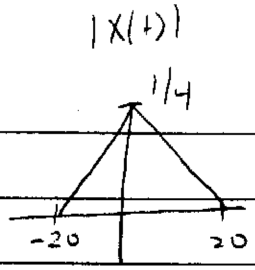
(d) $H(f) = 5\,\mathrm{sinc}(5f)$

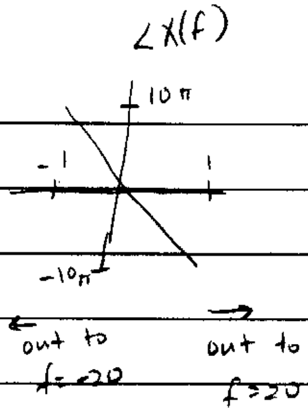
$$Y(f) = H(f)X(f) = 75\,\mathrm{sinc}^2(5f)\left(e^{-j2\pi 15/2 f} - e^{j2\pi 5/2 f}\right)$$

$$\overset{\mathcal{F}}{\longleftrightarrow}\ y(t) = 15\,\Lambda\left(\frac{(t - 15/2)}{5}\right) - 15\,\Lambda\left(\frac{(t + 5/2)}{5}\right)$$

2)

(a)

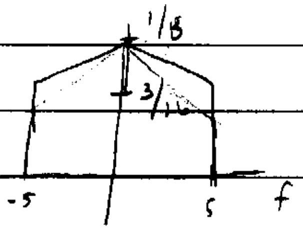




$$X(f) = \frac{1}{4}\Lambda(f/20)\, e^{-j2\pi 5f}$$

$$|X(f)| = 1/4\,\Lambda(f/20) \qquad \angle X(f) = -2\pi 5 f$$

(b) $H(f) = 1/2\,\mathrm{rect}(f/10)$



$$Y(f) = \begin{cases} 1/8\,\Lambda(f/20)\, e^{-j2\pi 5f}, & |f| \le 5 \\ 0, & \text{else} \end{cases}$$

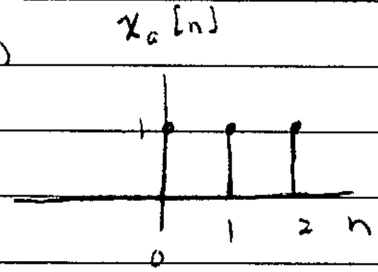
(c)

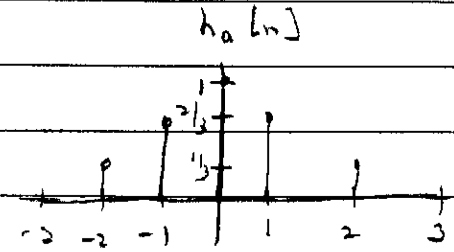
$$E_h = \int_{-\infty}^{\infty} |h(t)|^2\,dt = \int_{-\infty}^{\infty} |H(f)|^2\,df = \int_{-\infty}^{\infty} \left|\frac{1}{2}\mathrm{rect}(f/10)\right|^2 df$$

$$= \int_{-5}^{5} 1/4\,df = 10/4 = 5/2.$$

3)

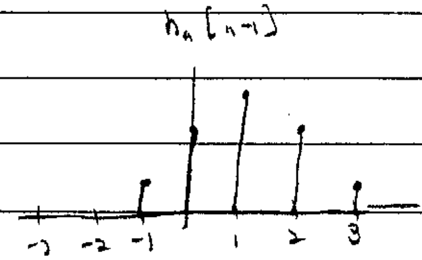
(a)

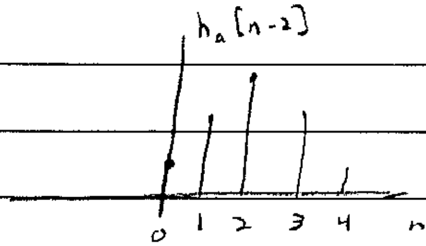




$$y_a[n] = h_a[n] * x_a[n]$$
$$= h_a[n] * [\delta[n] + \delta[n-1] + \delta[n-2])$$
$$= h_a[n] + h_a[n-1] + h_a[n-2]$$

$$y_a[n] = \begin{cases} 1/3, & n=-2,4 \\ 1, & n=-1,3 \\ 2, & n=0,2 \\ 7/3, & n=1 \\ 0, & \text{else} \end{cases}$$





(b) $y_b[n] = 3x_a[n] * (2h_a[n] + 3h_a[n-1]) = 6x_a[n] * h_a[n] + 9x_a[n] * h_a[n-1]$

$y_b[n] = 6y_a[n] + 9y_a[n-1]$

(c) $Y_c(\Omega) = -e^{-j\Omega}Y_c(\Omega) + X_c(\Omega)$

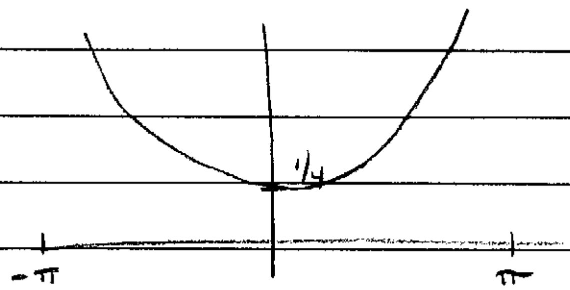
$$H(\Omega) = \frac{Y_c(\Omega)}{X_c(\Omega)} = \frac{1}{1+e^{-j\Omega}}$$

also okay to do $h[n] = (-1)^n$ and then $H(\Omega)$

$$|H(\Omega)|^2 = \frac{1}{1+e^{-j\Omega}} \cdot \frac{1}{1+e^{j\Omega}}$$

$$= \frac{1}{1+e^{j\Omega}+e^{-j\Omega}+1}$$

$$= \frac{1}{2+2\cos\Omega}$$



Blows up at π. $x_c(n) = (-1)^n$

4)

(a) $\Omega_0 = \pi/3$, gain at $\Omega = -\pi/3, \pi/3$ is $10/3$; thus

$y_a(n) = 50/3 \cos(\pi/3\, n)$

(b) $\Omega_0 = -2\pi/3$, gain at $\Omega = -2\pi/3, 2\pi/3$ is $20/3$; thus

$x_b[n] = 5\cos(2\pi/3\, n)$

$y_b[n] = 100/3 \cos(\pi/3\, n)$

(c) $x_c[n] = 5\cos(2\pi/3\, n)$

$\Rightarrow y_c[n] = y_b[n]$

(d)

$y_d(n) = y_a[n-2]$ (see (e))

(e) $H_2(\Omega) = H_1(\Omega)e^{-j\Omega 2}$

$\Rightarrow h_2[n] = h_1[n-2]$