ECE 313 - Signals and Systems, Fall 2012

Midterm Exam #1

Monday, October 15th, 7:00-9:00pm, ELABII 119

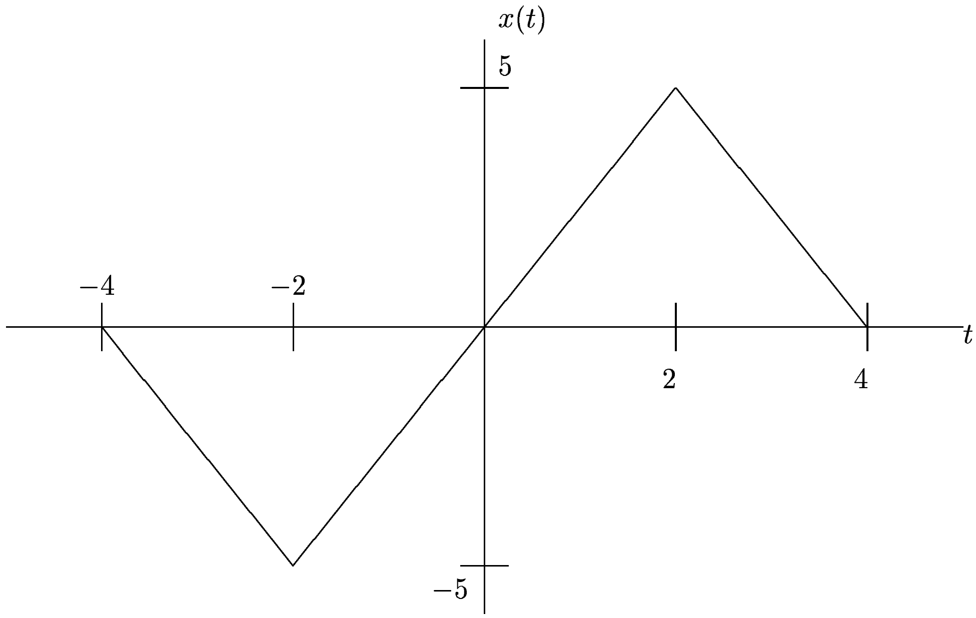
Overview

- The exam consists of five problems for 100 points. The points for each part of each problem are given in brackets **-** you should spend your **two hours** accordingly.
- The exam is closed book, but you are allowed **one page-side** of notes. Calculators are **not** allowed. I will provide all necessary blank paper.

Testmanship

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you stumble or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part, and, if possible, give the answer in terms of the quantities of the previous part that you are unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong (e.g. the Fourier Transform of a real signal that is not conjugate symmetric), be sure to write "this must be wrong because . . . " so that I will know you recognized such a fact.
- Academic dishonesty **will** be dealt with **harshly -** the *minimum penalty* will be an "F" for the course.

1. The signal $x(t)$ is shown below:



[8] (a) Write $x(t)$ in terms of the "triangle function" $\Lambda(t)$.

[5] (b) Find the Fourier transform $X(f)$ of $x(t)$.

[7] (c) The Fourier transform $X(f)$ can be written as $X(f) = a\ \text{sinc}^2(bf)\ \cos(2\pi cf + \theta)$ for the right choice of constants a, b, c, and θ. Find the values of a, b, c, and θ that make this true.

2. You are testing a new system at your company - call it "System A". You get the following results:

- When the input to System A is $u(t)$, the output is $\Lambda(t-1)$.
- When the input to System A is $u(t-1)$, the output is $\Lambda(t-2)$.
- When the input to System A is $2u(t)$, the output is $4\Lambda(t-1)$.
- When the input to System A is $2u(t) + u(t-1)$, the output is $4\Lambda(t-1) + \Lambda(t-2)$.

[5] (a) Is System A linear? ("yes", "no", "might be" are possible answers; a short justification is fine)

[5] (b) Is System A time-invariant? ("yes", "no", "might be" are possible answers; a short justification is fine)

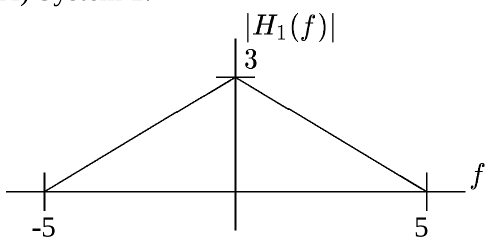
3. For a second system (System C), you instead have a characterization by an equation: $y(t) = \int_0^t x(\tau)d\tau$, where $x(t)$ is the input and $y(t)$ is the output.

[8] (a) Is System C linear? ("yes" and "no" are possible answers; if you say "yes", prove your answer from the definition of linearity. If you say "no", give a counterexample.)

[7] (b) Is System C time-invariant? ("yes" and "no" are possible answers; if you say "yes", prove your answer from the definition of time-invariance. If you say "no", give a counterexample.)

[5] (c) What is the output of System C when the input is $x(t) = 3\delta(t-2)$?

4. Consider the magnitude and phase of the frequency response $H_1(f)$ of a linear and time-invariant (LTI) System 1:

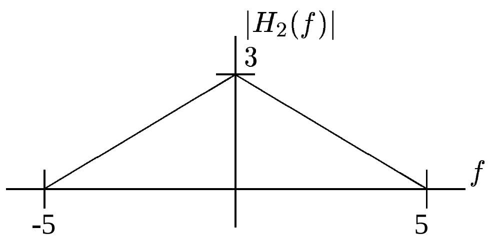


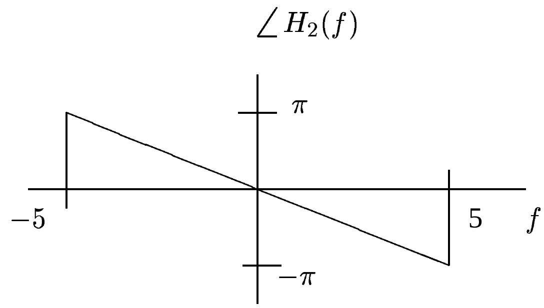
phase $H_1(f) = 0$ for all f

[5] (a) Suppose $x_1(t) = 5\cos(2\pi\ 2.5\ t)$ is input to System 1. Find the corresponding output $y_1(t)$ of System 1.

[5] (b) Suppose $x_1(t) = 5\cos(2\pi\ 2.5\ t)$ is input to System 1. Find the power in the output $y_1(t)$.

Consider the frequency response $H_2(f)$ of a linear and time-invariant (LTI) System 2:





[10] (c) Suppose the signal $x_2(t) = 5\sin(2\pi\ 2.5\ t) + 10\cos(2\pi 10t)$ is input to System 2. Find the corresponding output $y_2(t)$ of the system.

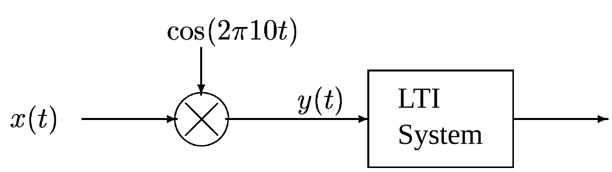
5. A form of amplitude modulation (AM) radio multiples a message $m(t)$ by a carrier $\cos(2\pi f_c t)$, where f_c is the "carrier frequency" (which is the number to which you dial the radio in your car to hear that station). Suppose our message is $m(t) = \text{sinc}(4t)$ and the carrier frequency is $f_c = 10$ Hz. Let's use our ECE 313 knowledge to analyze an AM transmitter and design a receiver.

   [10] (a) Find **and sketch** the Fourier transform of $x(t) = \text{sinc}(4t)\cos(2\pi 10t)$. (*Hint: One method is to use the appropriately named "modulation property".*)

   [5] (b) Find the energy in $x(t)$.

   [5] (c) Is your AM transmitter with input $m(t)$ and output $x(t)$ a linear time-invariant (LTI) system?

   [10] (d) Finally, let's consider how to build the receiver, the goal of which is to take in $x(t)$ and produce $m(t)$. One method for doing such is to multiply $x(t)$ by $\cos(2\pi 10t)$ to form $y(t) = x(t)\cos(2\pi 10t)$ and then apply a linear time-invariant (LTI) system to $y(t)$ to leave only $Am(t)$, where A is some positive constant that we do not care about:



   - Find **and sketch** $Y(f)$, the Fourier transform of $y(t)$.
   - Draw the frequency response $H(f)$ of a LTI system such that, when $y(t)$ is input to the system, $Am(t)$ comes out for some constant $A > 0$ of your choice.