ECE 317 Homework & Solutions

(a) $x(t) = 5 \sin^2(5t) + \cos(20\pi ft)$

$$X(f) = \frac{1}{2} \left( \delta(f - 5) + \delta(f + 5) \right)$$

After sampling at 10 Hz:

Since this adds a delta at 0 Hz, low-pass filtering cannot recover the original $X(f)$.

(b) After sampling at 20 Hz:

To recover the original signal, we need to apply a filter $H(f)$ with frequency response $H(f) = 1$ for $|f| \leq 10$ Hz, and $H(f) = 0$ for $|f| > 10$ Hz. Example:

Then $X_s(f) \cdot H(f) = X(f)$ and we recover the original signal.

(c) $x(t) = 5 \sin^2(5t) + \sin(20\pi ft)$

$$X(f) = \frac{1}{2} \left( \delta(f - 10) + \delta(f + 10) \right)$$

$X(f)$
After sampling at 20 Hz, the positive and negative deltas cancel each other:

\[ X(f) \]

So the original signal cannot be recovered.

(d) After sampling at 21 Hz:

\[ X(f) \]

So we can recover the original signal \( x(t) \) by applying a filter \( h(t) \) with frequency response \( H(f) = 1 \) for \( |f| < 10 \) and \( H(f) = 0 \) for \( |f| > 11 \). An example is the perfect low-pass filter \( H(f) = \text{rect}(f/21) \).

(b) \( x(t) \) can be reconstructed from the samples by applying a perfect low-pass filter \( h(t) \) with frequency response \( H(f) = \text{rect}(f/60) \).

(b) After sampling at 10 Hz:

\[ X(f) \]

Prof. Goechel is correct: to recover, we need to run the sampled signal through a band-pass filter \( h(t) \) with frequency response \( H(f) = \frac{1}{2} \) for \( 20 < |f| < 30 \), and \( H(f) = 0 \) elsewhere. \( H(f) \)
(c) For $y(t)$, the sampled spectrum is as follows:

Therefore, he can reconstruct $y(t)$ from the samples using a bandpass filter $h(t)$ with frequency response $H(f) = \frac{1}{2}$ for $15 < |f| \leq 25$ and $H(f) = 0$ for $|f| \leq 5$, $35 < |f| \leq 45$, $55 < |f| \leq 65$, ... An example is the perfect bandpass filter $H(f) = \text{sinc} \left( \frac{f-20}{10} \right) + \text{sinc} \left( \frac{f+20}{10} \right)$.

3. The spectrum of the sampled signal is as follows: copies are placed every $f + f_s$

The sum of all the copies gives the spectrum $X_s(f)$

So $X(t) = \delta(t)$ one there is one sample equal to one at time zero; all other samples are equal to zero.

4. The apparent frequency is $f_a = f - m f_s$ where $m$ is the integer that gives 

\[-f_s/2 \leq f_a \leq f_s\]. $f_s = 20$ kHz and $f_s/2 = 10$ kHz. 

(a) $f_0 = 8$ kHz; letting $m = 0$ gives $f_a = 8 - 0 \cdot 20 = 8 < 10$, so $f_a = 8$ kHz. 

(b) $f_0 = 12$ kHz; letting $m = 1$ gives $f_a = 12 - 1 \cdot 20 = -8 > -10$, so $f_a = 8$ kHz and the phase changes.

(c) $f_0 = 20$ kHz; letting $m = 1$ gives $f_a = 20 - 1 \cdot 20 = 0$, so $f_a = 0$ kHz. 

(d) $f_0 = 22$ kHz; letting $m = 1$ gives $f_a = 22 - 1 \cdot 20 = 2 > 0$, so $f_a = 2$ kHz. 

(e) $f_0 = 32$ kHz; letting $m = 2$ gives $f_a = 32 - 2 \cdot 20 = 8 > 0$, so $f_a = 8$ kHz, and the phase changes.
(5) \( x(t) = 3 \cos(5t) + \cos(16t) + 2 \cos(20t) \) sampled at rate 25% above Nyquist.

(a) What is Nyquist rate? \( X(f) = \frac{3}{2}(\delta(f-5) + \delta(f+5)) + \frac{1}{2}(\delta(f-8) + \delta(f+8)) \\
+ \delta(f-10) + \delta(f+10) \)

Highest frequency is 10 Hz, so Nyquist rate is 20 Hz and \( f_s = 1.25 \times 20 = 25 \) Hz.

Sampled signal spectrum: \( x_s(f) \)

To reconstruct \( x(t) \) from the sampled signal, we run it through a low-pass filter \( h(t) \) with \( H(f) = 1 \) for \( |f| < 10 \) and \( H(f) = 0 \) for \( |f| > 15 \).

For example: perfect LFT

(b) If \( f_s = 0.75 \times 20 = 15 \) Hz, the cutoff frequency is \( \frac{f_s}{2} = 7.5 \) Hz.

The apparent frequencies are: (using the equation from Problem 4)

For 3 Hz: letting \( m = 0 \) gives \( f_a = 3 - 0.15 = 2.85 \) Hz \( f_a = 3 \) Hz

For 8 Hz: letting \( m = 1 \) gives \( f_a = 8 - 1.15 = 6.85 \) Hz so \( f_a = 7 \) Hz

For 10 Hz: letting \( m = 1 \) gives \( f_a = 10 - 1.15 = 8.85 \) Hz so \( f_a = 9 \) Hz

So the frequencies at the output are 3 Hz, 7 Hz, 9 Hz.

(6) \( p(t) = \text{sinc}(t/4) = \text{sinc}(4t) \). It is the only signal that is bandlimited to \( 2f_a \) Hz with these sample values.