ECE 313 Homework #9 - Due 12/07/2012 at 10:10am

1. Consider the system in Figure 1 to process continuous-time signals with discrete-time processing. Note that there is no anti-aliasing filter on the front-end of the sampler. (15 points for each part)

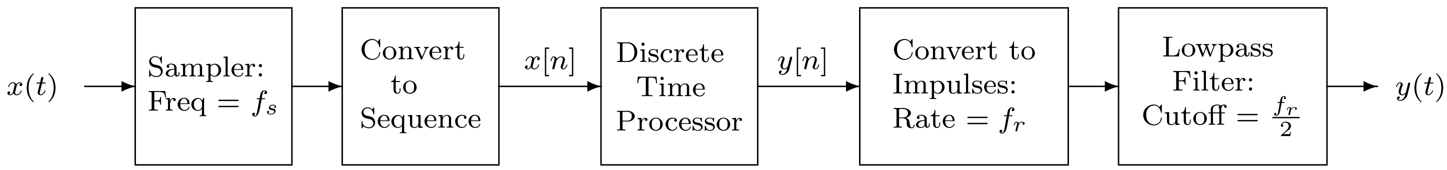

Figure 1

For parts (a-b), suppose the input to the system is given by $x(t) = 20000\mathrm{sinc}(20000\mathrm{t})$.

(a) Suppose that $f_s = f_r = 25$ KHz, and the discrete-time processor is the filter with frequency response

$$H_{DT}(\Omega) = 1 - \frac{|\Omega|}{\pi}, \quad -\pi < \omega \leq \pi,$$

extended to be periodic with period 2π. Find the Fourier transform $Y(f)$ of the output $y(t)$.

(b) Suppose that $f_s = f_r = 15$ KHz, and there is no discrete-time processing (i.e., $y[n] = x[n]$). Find the Fourier transform $Y(f)$ of the output $y(t)$.

For parts (c-d), suppose the input to the system is given by $x(t) = \cos(2\pi 3000t)$.

(c) Suppose that $f_s = 12$ KHz, $f_r = 24$ KHz, and that the discrete-time processor is an interpolator by a factor of 2 (which consists of a "zero insertion" followed by a lowpass filter). Find the output $y(t)$.

(d) Suppose that $f_s = f_r = 12$ KHz and that the discrete-time processor is an interpolator by a factor of 2. Find the output $y(t)$.

2. You are an intern in a communications company. Your supervisor directs you to use a receiver to collect a radar signal $x(t)$ with Fourier transform shown in Figure 2. The receiver samples the signal at a rate of 25 KHz to produce a discrete-time sequence $x[n]$ that you save in a file.

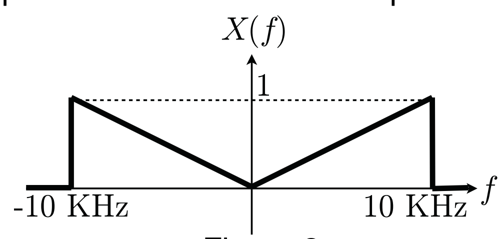

Figure 2

The next day, your supervisor points out that he forgot that the signal $x(t)$ needed to be lowpass-filtered using the filter $H_{\mathrm{des}}(f)$ in Figure 3 before sampling, and is asking you to solve this problem.

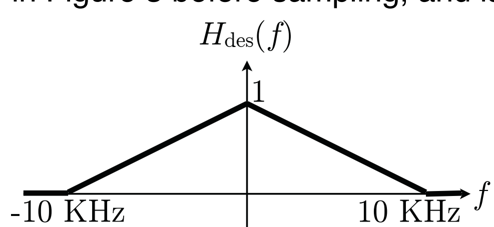

Figure 3

Notation: $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$, $\mathrm{rect}(t) = 1$ if $|t| \leq 1/2$, and $\mathrm{rect}(t) = 0$ otherwise.

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The original waveform is no longer available; all you have are the samples you saved. A friend provides you with a discrete-time filter having frequency response $H_{\mathrm{DT}}(\Omega)$ given in Figure 4.

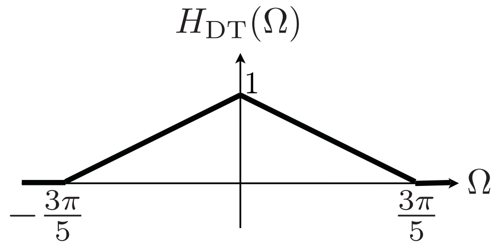

Figure 4

Give a block diagram for processing the samples $x[n]$ to obtain the filtered sequence $y[n]$ requested by your supervisor, which corresponds to the samples of $y(t) = x(t) * h_{\mathrm{des}}(t)$ at 25 KHz. Available blocks include upsampling, downsampling, and perfect low-pass filter blocks, together with a block for the available filter $H_{\mathrm{DT}}(\Omega)$. (25 points)

3. Consider a system that has two continuous-to-discrete converters running at a frequency f_s, a discrete-time processor, and a discrete-to-continuous converter running at frequency f_r:

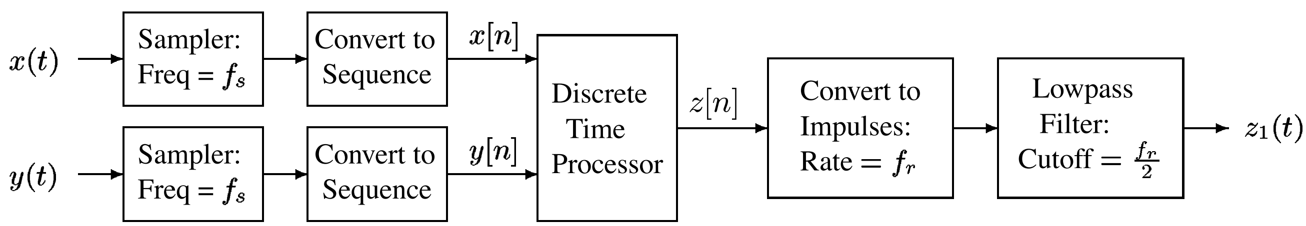

Figure 5

As input, we have two signals $x(t)$ and $y(t)$ each of bandwidth 10 KHz. Find the lowest values for f_s and f_r and the necessary discrete-time processor block such that $z_1(t) = x(t) + y(t)$. (15 points)

4. **[Extra problem, no credit]** Consider once again the system in Figure 1. Using such a block diagram, you want to build a low-pass filter $H(f) = \mathrm{rect}(f/30000)$, to be applied on input signals $x(t)$ of bandwidth up to 25 KHz to yield the signal $y(t)$. Find the lowest values of f_s and f_r and the discrete-time processor $H_{\mathrm{DT}}(\Omega)$ that can implement this continuous-time filter.

Notation: $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$, $\mathrm{rect}(t) = 1$ if $|t| \leq 1/2$, and $\mathrm{rect}(t) = 0$ otherwise.