ECE 313 Homework #8 - Due 11/30/2012 at 10:10am

1. Consider the sampling and reconstructions setups below. (5 points each)

(a) A signal $x(t) = 5\mathrm{sinc}^2(5t) + \cos(20\pi t)$ is sampled at a rate of 10 Hz. Find the spectrum of the sampled signal. Can $x(t)$ be reconstructed by lowpass filtering the sampled signal?
(b) Repeat part (a) for a sampling frequency of 20 Hz. Can you reconstruct the signal $x(t)$ from this sampled signal? Explain.
(c) If $x(t) = 5\mathrm{sinc}^2(5t) + \sin(20\pi t)$, can you reconstruct $x(t)$ from its samples at a rate of 20 Hz? Explain your answer with spectral representations.
(d) For $x(t) = 5\mathrm{sinc}^2(5t) + \sin(20\pi t)$, can you reconstruct $x(t)$ from its samples at a rate of 21 Hz? Explain your answer with spectral representations. Comment on your results.

2. It turns out that, in certain cases, Nyquist is pessimistic on the achievable lowest sampling rate.

(a) The highest frequency in the spectrum $X(f)$ in Figure 1 of a bandpass signal $x(t)$ is 30 Hz. Hence, the minimum sampling frequency needed to sample $x(t)$ is 60 Hz. Can you reconstruct $x(t)$ from these samples? How? (5 points)

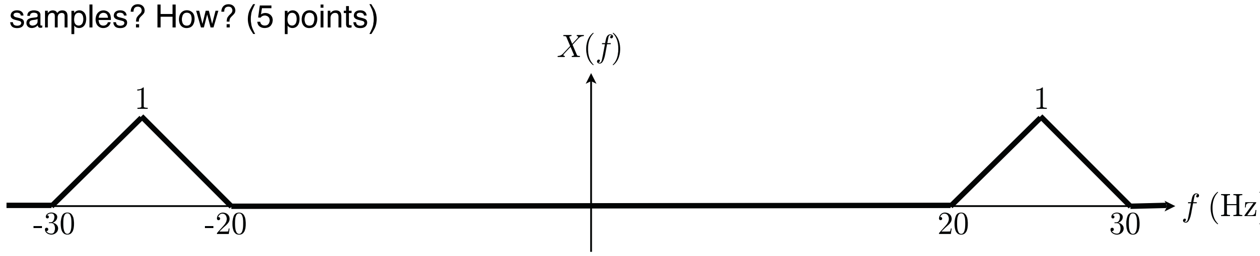

Figure 1

(b) Prof. Goeckel looks at $X(f)$: he concludes that its bandwidth is really 10 Hz, and decides that the sampling rate 20 Hz is adequate for sampling $x(t)$. Sketch the spectrum of the signal sampled at a rate of 20 Hz. Is Prof. Goeckel correct? If yes, how can he reconstruct the signal $x(t)$ from these samples? (10 points)
(c) Prof. Duarte, using Prof. Goeckel's reasoning, looks at $Y(f)$ in Figure 2, the spectrum of another bandpass signal $y(t)$, and concludes that he can use a sampling rate of 20 Hz to sample $y(t)$. Sketch the spectrum of the signal $y(t)$ sampled at a rate of 20 Hz. Can he reconstruct $y(t)$ from these samples? How? (10 points)

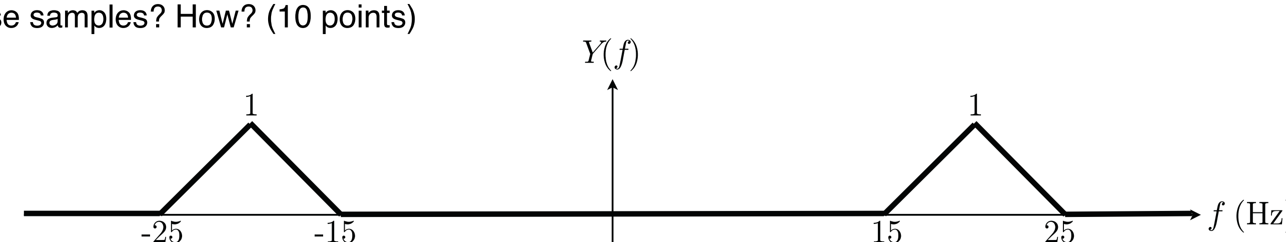

Figure 2

Notation: $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.

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3. A signal $x(t)$, whose spectrum $X(f)$ is shown in Figure 3, is sampled at a rate of $f_s = f_1 + f_2$ Hz. Find all the sample values of $x(t)$ merely by inspection of $X(f)$. (15 points)

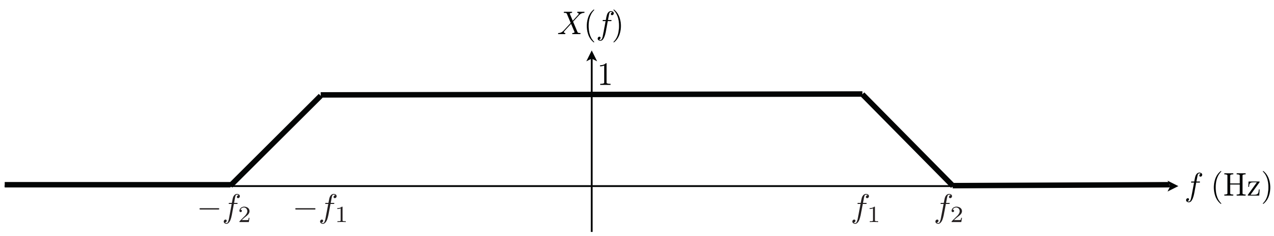

Figure 3

4. A sinusoid of frequency f_0 Hz is sampled at a rate $f_s = 20$ Hz. Find the apparent frequency of the sampled signal if f_0 is (a) 8 Hz, (b) 12 Hz, (c) 20 Hz, (d) 22 Hz, (e) 32 Hz. (4 points each)

5. A signal $x(t) = 3\cos(6\pi t) + \cos(16\pi t) + 2\cos(20\pi t)$ is sampled at a rate 25% above Nyquist rate.

(a) Sketch the spectrum of the sampled signal. How would you reconstruct $x(t)$ from these samples? (10 points)

(b) If the sampling frequency is 25% below the Nyquist rate, what are the frequencies of the sinusoids present in the output of the filter with cutoff frequency equal to the half the sampling frequency? Do not write the actual output; give just the frequencies of the sinusoids present in the output. (10 points)

6. **[Extra problem, no credit]** In digital communication systems, the efficient use of channel bandwidth is ensured by transmitting digital data encoded by means of bandlimited pulses. Unfortunately, bandlimited pulses are non-time-limited; that is, they have infinite duration, which causes pulses representing successive digits to interfere and cause errors in the reading of true value pulses. This difficulty can be resolved by shaping a pulse $p(t)$ in such a way that it is bandlimited, yet causes zero interference at the sampling instants. To transmit R pulses per second, we require a minimum bandwidth $R/2$ Hz. The bandwidth of $p(t)$ should be $R/2$ Hz, and its samples, in order to cause no interference at all other sampling instants, must satisfy the condition

$$p(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \quad T = \frac{1}{R}$$

Because the pulse rate is R pulses per second, the sampling instants are located at intervals of $1/R$ seconds. Hence, the foregoing condition ensures that any given pulse will not interfere with the amplitude of any other pulse at its center. Find $p(t)$. Is $p(t)$ unique in the sense that no other pulse satisfies the given requirements?

Notation: $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.