ECE 313 Homework #6 - Due 10/31/2012 at 10:10am

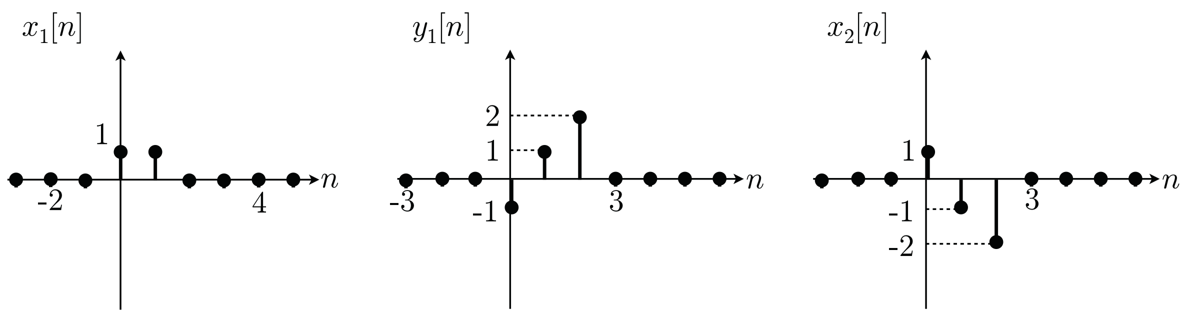
1. The fact that operational amplifiers (op-amps) in practice are non-linear devices may be scary at first, but the Fourier transform greatly simplifies the analysis of their response to sinusoids. Assume you are given an op-amp with transfer function $y(t) = 10x(t) + 0.01x^3(t)$.

   (a) Is the amplifier an LTI system? (2 points)
   (b) Assume you use an input $x(t) = A\cos(2\pi f_c t)$. Find the value of the output $y(t)$ and its Fourier transform $Y(f)$. (3 points)
   (c) The harmonics of the original sinusoidal input $x(t)$ correspond to additional components in the output at frequencies that are multiples of the input frequency $2f_c, 3f_c, \ldots$ Identify the frequencies and magnitudes of any harmonics that appear in the output $y(t)$. (5 points)
   (d) An operational amplifier is said to saturate (or to operate in its nonlinear region) if A is very large (e.g., $A = 100$). How do the output harmonics' amplitudes compare against that of the fundamental frequency at saturation? What if A is small (e.g., $A = 1$)? (5 points)

2. Classify the following systems as to linearity, time-invariance, causality, and stability. (10 points each)

   (a) $y[n] = \sum_{m=-\infty}^{n+1} x[m]$ (b) $y[n] = x[n]/x[n+1]$ (c) $y[n] = \sqrt{x[n]}$ (d) $y[n] = e^{j\pi n/2}x[n-1]$
   (e) $y[n] = x[n]r[n]$ (f) $y[n] = \frac{1}{2}\sum_{k=-\infty}^{\infty} x[k](\delta[n-k] + \delta[n+k])$ (g) $y[n] = \alpha^n x[n]$

3. A linear, time-invariant system produces an output $y_1[n]$ in response to an input $x_1[n]$ as shown below. Determine and sketch the output $y_2[n]$ that results when input $x_2[n]$ is applied to the same system. (15 points)



4. **[Extra problem, no credit]** A moving average is used to detect a trend in a rapidly fluctuating variable such as the price of a stock. A variable may fluctuate (up and down) daily, masking its long-term (secular) trend. We can discern the long-term trend by smoothing or averaging the past N values of the variable. For a stock price, we may consider a $N = 5$-day moving average $y[n]$ for day n as the mean of the past 5 day's market closing value $x[n]$.
   (a) Write the system equation that links $x[n]$ and $y[n]$.
   (b) Is this system linear, time-invariant, stable, causal?
   (c) The company Raised Sine Inc. can set its stock's closing value to be $x[n] = 1 + \sin(2\pi f_c n)$, and they want to be able to report no fluctuation in its moving average $y[n]$. What should they set f_c to?

Notation: $r[n] = \begin{cases} n & \text{if } n >= 0 \\ 0 & \text{if } n < 0 \end{cases}$ is the discrete ramp function.