Exam 1 Solutions

ECE 313 Fall, 2012

1) (a)

$$x(t) = 5\Lambda\left(\frac{(t-2)}{2}\right) - 5\Lambda\left(\frac{(t+2)}{2}\right)$$

(b) scaling/shifting

$$X(f) = 5 \cdot 2\,\mathrm{sinc}^2(2f)\,e^{-j2\pi f 2}$$

$$- 5 \cdot 2\,\mathrm{sinc}^2(2f)\,e^{j2\pi f 2}$$

$$= 10\,\mathrm{sinc}^2(2f)\cdot\left(e^{-j2\pi f 2} - e^{+j2\pi f 2}\right)$$

(c)

$$X(f) = 10\,\mathrm{sinc}^2(2f)\cdot 2j\left(\frac{1}{2j}e^{-j2\pi f 2} - \frac{1}{2j}e^{+j2\pi f 2}\right)$$

$$= 20j\,\mathrm{sinc}^2(2f)\left(-\sin(2\pi 2f)\right)$$

$$= -20j\,\mathrm{sinc}^2(2f)\cos(2\pi 2f - \pi/2)$$

$a = -20j$, $b = 2$, $c = 2$, $\theta = -\pi/2$

(lots of other correct answers also: $\theta = 3\pi/2$
-or- you could use $a = 20j$, and $\theta = \pi/2$ or $\theta = -3\pi/2$, etc.)

2) (a) no. If the response to $u(t)$ is $\Lambda(t-1)$, the response of a linear system to $2u(t)$ is $2\Lambda(t-1)$.

(b) might be The responses to $u(t)$, $u(t-1)$ agree with the TI definition. But TI requires it to be true for all inputs, all shifts.

3)

(a) yes

$$x_1(t) \rightarrow \boxed{} \rightarrow y_1(t) = \int_0^t x(\tau)\,d\tau$$

$$x_2(t) \rightarrow \boxed{} \rightarrow y_2(t) = \int_0^t x(\tau)\,d\tau$$

$$\alpha_1 x_1(t) + \alpha_2 x_2(t) \rightarrow \boxed{} \rightarrow \int_0^t \left(\alpha_1 x_1(\tau) + \alpha_2 x_2(\tau)\right) d\tau$$

$$= \alpha_1 \int_0^t x_1(\tau)\,d\tau + \alpha_2 \int_0^t x_2(\tau)\,d\tau$$

$$= \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

(b) no

counterexample

$$x_1(t) = \delta(t+1) \implies y_1(t) = 0$$

$$x_2(t) = \delta(t-1) \implies y_2(t) = u(t-1) \neq y_1(t-2)$$

(many other possibilities of course)

(c) $$y(t) = \int_0^t 3\delta(\tau-2)\,d\tau = 3 \underbrace{\int_0^t \delta(\tau-2)\,d\tau}$$

$$= \begin{cases} 1, & \text{if } 2 \in [0,t] \\ 0, & \text{else} \end{cases} = \begin{cases} 1, & t \in [2,\infty) \\ 0, & \text{else} \end{cases}$$

$$\implies y(t) = u(t-2)$$

4)

(a)

$$X_1(f) = \frac{5}{2}\delta(f-2.5) + \frac{5}{2}\delta(f+2.5)$$

$$Y_1(f) = H_1(f)X_1(f) = \frac{15}{4}\delta(f-2.5) + \frac{15}{4}\delta(f+2.5)$$

$$\Rightarrow y_1(t) = \frac{15}{2}\cos(2\pi\, 2.5\, t)$$

(b) $P = \frac{A^2}{2} = \frac{225}{4} \cdot \frac{1}{2} = 225/8$

(c)

$$X_2(f) = \frac{5}{2j}\delta(f-2.5) - \frac{5}{2j}\delta(f+2.5) + 5\,\delta(f-10) + 5\,\delta(f+10)$$

$$Y_2(f) = H(f)\,X_2(f) = \frac{5}{2j} \cdot 3/2 \cdot \overset{-j}{\cancel{e^{-j\pi/2}}}\,\delta(f-2.5)$$

$$- \frac{5}{2j} \cdot 3/2 \cdot \overset{j}{\cancel{e^{j\pi/2}}}\,\delta(f+2.5)$$

$$= -15/4\,\delta(f-2.5) - \frac{15}{4}\delta(f+2.5)$$

$$= -15/2\cos(2\pi\, 2.5\, t)$$

5)

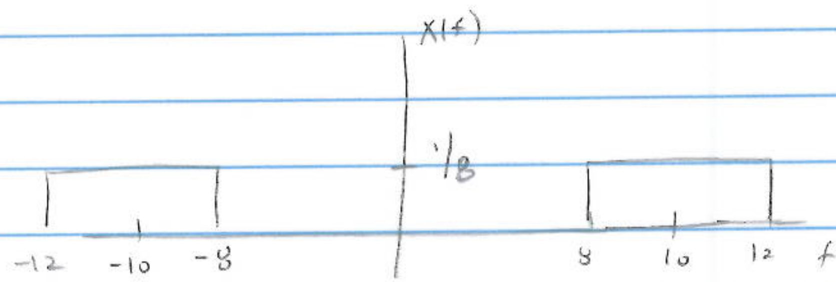
(a)

$$x(t) = \text{sinc}(4t)\cos(2\pi 10 t)$$

$$\Rightarrow X(f) = \mathcal{F}\{\text{sinc}(4t)\} * \mathcal{F}\{\cos(2\pi 10 t)\}$$

$$= \frac{1}{4}\text{rect}(f/4) * \left(\frac{1}{2}\delta(f-10) + \frac{1}{2}\delta(f+10)\right)$$

$$= \frac{1}{8}\text{rect}\left(\frac{(f-10)}{4}\right) + \frac{1}{8}\text{rect}\left(\frac{(f+10)}{4}\right)$$



(b)

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} |X(f)|^2\,df$$

remember to square 1/8

$$= 2 \cdot \int_{8}^{12} 1/64\,df$$

$$= 2 \cdot 4/64 = 1/8$$

(c) If LTI, I can find an H(f).

Try $H(f) = Y(f)/X(f) \Rightarrow$ divide by zero at $f \notin [8, 12]$

<u>No</u>.

$$y(t) = x(t)\cos(2\pi\, 10t)$$

$$= \text{sinc}(4t)\cos^2(2\pi 10 t)$$

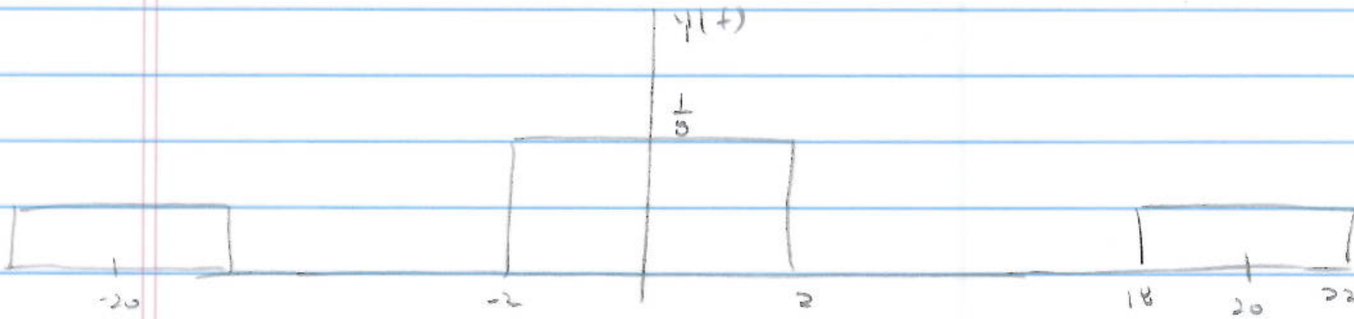
can also do

$$Y(f) = X(f) * \left(\tfrac{1}{2}\delta(f-f_0) + \tfrac{1}{2}\delta(f+f_0)\right)$$

$$= \text{sinc}(4t)\cdot\left(\frac{1}{2} + \frac{1}{2}\cos(2\pi 20t)\right)$$

$$= \frac{1}{2}\text{sinc}(4t) + \frac{1}{2}\text{sinc}(4t)\cos(2\pi 20t)$$

$$= \frac{1}{8}p(f/4) + \frac{1}{60}p\left(\frac{(f-20)}{4}\right) + \frac{1}{16}p\left(\frac{(f+20)}{4}\right)$$



Let $H(f) = \begin{cases} 1, & |f| \le 2 \\ 0, & \text{else} \end{cases}$

Then

$$H(f)Y(f) = \frac{1}{8}p(f/4)$$

and the output is

$$\mathcal{F}^{-1}\{1/8\, p(f/4)\} = 2\,\text{sinc}(4t) = 2\,x(t)$$