Time-Selective Sampling Receiver for Interference Rejection

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Abstract—The use of time-selective sampling is presented as a way to significantly reduce the effect of large interferers on the sensitivity of receivers operating in a crowded spectrum. The concept assumes a discrete-time analog front end, where large distorted samples are discarded and small undistorted samples are retained for use in estimating the desired message signals. Since the retained samples are not uniform in time, signal processing is a challenge. The signal processing is described in cases in which 1) the frequencies of the interferers are known and 2) the frequencies are not known.

Index Terms—Cognitive radio, interference, interference suppression, intermodulation distortion, nonlinear distortion, nonuniform sampling software-defined radio, sampling receiver.

I. INTRODUCTION

T

HE ideal receiver of the future will be capable of collecting wide swaths of spectrum and then adaptively choosing which parts of it to extract. Such receivers would be necessary to implement the techniques of dynamic spectrum access and cognitive radio [1], [2]. To be fully adaptable, their radiofrequency (RF) stages would minimize fixed filtering, while maintaining the capability of extracting very weak signals in the presence of signals that may be more than 90 dB larger [2].

Normally, the RF input to a receiver passes through a fixed prefilter to remove interferers and is then amplified by a low-noise amplifier (LNA) to boost the signal level above the noise generated in succeeding stages (reconfigurable filters, samplers, and downconverters.) To permit adaptability, the fixed filter must be removed. This makes the receiver susceptible to interferers that mix in the LNA nonlinearities to generate intermodulation products that may then mask a small desired signal. A tunable prefilter can be used, but this adds complexity and loss.

Instead of filtering in the frequency domain, we proposed [3] postfiltering in the time domain. In such a receiver, the received signal is sampled after the LNA, and all samples larger than some threshold are discarded under the premise that they have been distorted by the LNA nonlinearity. The retained samples have benefited by the gain of the LNA, without being distorted by its nonlinearity. Their sample times depend on the interferer zero crossings and are, therefore, not uniform. Reconstructing a signal from a nonuniformly sampled

data set has been studied in connection with digital audio [4], geophysics, and radio astronomy. In a related manner, we herein directly extract the magnitude and phase variation of a small message signal from low-distortion samples that contain both the message and the interference.

In a conference brief [3], we described the basic time-selective sampling idea and presented measurements of a simple test case. In Section II, we review the concept. The basic contributions of this brief are described in Sections III and IV, where the message retrieval algorithms are described for cases where the interferer frequencies are known and not well known, respectively. We discuss the effect of noise when too few samples are retained versus distortion when too many are retained.

II. BASIC CONCEPT

The input–output relation that approximates a memoryless amplifier below hard saturation is

\[ v_o(t) = k_1 v_i(t) + k_2 v_i(t)^2 + k_3 v_i(t)^3 \]  

(1)

where \( v_i \) is the amplifier input voltage, and \( v_o \) is its output. \( k_1 \) is determined by the linear gain of the amplifier. \( k_2 \) and \( k_3 \) depend on the amplifier’s second- and third-order intercepts. The second-and third-order terms add distortion at times when \( v_i \) is large. In the proposed receiver, the output of the LNA is sampled, but the samples with magnitude greater than some threshold \( V_T \) are discarded. The result is a set of sample voltages corresponding to a set of sample times \( \{ t_n \} \). These samples have gone through a nearly linear amplification since they are small enough to make the nonlinear terms in (1) negligible. Fig. 1 shows a possible output, with the retained sample points marked. Note that, as the interferers become
larger, there are fewer samples small enough to accept, and this will make it more difficult to detect a message symbol accurately. The accepted samples contain both the interferer and the message, but are not distorted.

In the worst case, the message is much smaller than the interferer. The small message signal is recovered by running at a rate \( \tau \) much larger than \( V_T \), so that, in a block of time \( T \), there are many clock cycles between samples, and some cases where many samples will be closely spaced. An integrator is included to accumulate samples (relatively few) over time \( T \), in order to reduce the required analog-to-digital conversion (ADC) rate. The receiver in Fig. 2 passes to a digital signal processor (DSP) both the integrated samples and a clock pulse when an acceptable sample is taken. In addition, information as to whether the signal \( v_o \) is greater than \( V_T \) or less than \( -V_T \) could be also passed; such information has been shown to aid in sparse signal recovery [4]–[6].

III. FORMULATION: KNOWN INTERFERER SPECTRUM

Signal recovery based on nonuniform sampling has been studied for decades [7]–[11]. Researchers have investigated sampling at user-prescribed pseudorandom times, in order to reduce aliasing effects [10] and in connection with sparse signal assumptions [14]. In contrast to these applications, the location of sample times in our problem depends on the interference.

In what follows, the small message signal is recovered by a least mean square (LMS) fit of a signal expansion to the samples sent to the DSP by the ADC in Fig. 2. The formulation recovers the signal’s spectral amplitudes and phase directly without reconstituting the full time response. In this section, we assume that the spectral supports of the interferer and message are known.

A. Basic Analysis

Referring to Fig. 2, we assume a uniform sample clock running at a rate \( f_s \), such that, in a block of time \( T\text{dur} \), the total number of clock pulses is \( N_P = T\text{dur} f_s \). When the output signal has amplitude \( \max(|v_o|) > V_T \), only \( N_S < N_P \) samples out of that block are passed on to the integrator. Therefore, \( N_S \) is the number of samples in the set \( \Lambda = \{ t_n \} \). The vector \( \mathbf{v}_o = (v_o(t_n)) \) is the \( N_S \times 1 \) vector of retained samples.

We also divide the total block time into intervals of duration \( T_{ADC} \), each with \( N_I \) sample slots. There would be \( T_{dur}/T_{ADC} = N_P/N_I \) such intervals. The retained samples in each interval are summed and the output sent to the ADC.

The operation that places the \( N_S \) samples in \( v_o \), into \( M_I \) integrations that make up \( \mathbf{v}_{ADC} \) can then be described by

\[
\mathbf{v}_{ADC}(m) = \sum_{n=1}^{N_S} g(t_n - m T_{ADC}) \cdot \mathbf{v}_o(t_n)
\]

where \( g(\tau) = 1 \) for \( 0 < \tau < T_{ADC} \) and is zero otherwise. (2) can be written in vector form as \( \mathbf{v}_{ADC} = \mathbf{G} \cdot \mathbf{v}_o \), where \( \mathbf{G} \) is an \( M_I \times N_S \) matrix. If every interval had at least one retained sample, then \( M_I = N_P/N_I \). Some of the integration intervals will have every sample slot filled, some may have very few filled, and some may have zero samples, depending on \( V_T \), \( \max(|v_o|) \), and \( T_{ADC} \). In forming \( \mathbf{G} \), we eliminate every row corresponding to an interval with zero samples, and thus, \( M_I \leq N_P/N_I \). \( \mathbf{G} \) is formed in the DSP from the known summation index \( N_I \) and the threshold-conditioned clock signal shown in Fig. 2.

In order to recover the entire undistorted output voltage, we assume that it can be represented by a set of sinusoids made up of \( \omega_n \) plus a dc term, and \( t_m \in \Lambda \). Thus

\[
v_{oE}(t_m) = c_0 + \sum_{n=1}^{N_I} \left[ c_{Rn} \cos(\omega_n t_m) - c_{In} \sin(\omega_n t_m) \right].
\]

In matrix form, (3) is

\[
v_{oE} = \mathbf{F} \cdot \mathbf{x} = [\mathbf{F}_c, \mathbf{F}_s] \cdot \mathbf{x},
\]

where the real vector \( \mathbf{x} = [c_0, c_{R1}, c_{R2}, \ldots, c_{RN_I}, c_{I1}, c_{I2}, \ldots, c_{IN_I}]^T \) contains the in-phase and quadrature parts of the \( N_f \) spectral components. The matrix \( \mathbf{F} \) has dimension \( N_S \times (2N_f + 1) \) and is made up of a concatenation of a unit column vector and two matrices

\[
(F_c)_{nm} = \cos(\omega_n t_m) \text{ and } (F_s)_{nm} = -\sin(\omega_n t_m).
\]

The spectral coefficients \( \mathbf{x} \) are estimated by LMS fitting \( \mathbf{v}_{oE} \) to \( \mathbf{v}_o \), subject to the integration matrix \( \mathbf{G} \) and a diagonal weighting matrix \( \mathbf{W} \), i.e.,

\[
\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \| \mathbf{W} \cdot (\mathbf{v}_o - \mathbf{v}_{oE}) \|_2
\]

\[
= \arg \min_{\mathbf{x}} \| \mathbf{W} \cdot (\mathbf{v}_{ADC} - \mathbf{G} \cdot \mathbf{F} \cdot \mathbf{x}) \|_2
\]

where each weight in \( \mathbf{W} \) is the inverse of the number of valid samples in the corresponding integration interval. The integration operation \( \mathbf{G} \) reduces the number of measurements in this system from \( N_S \) to \( M_I \). We define a measurement matrix \( \mathbf{A} = \mathbf{W} \cdot \mathbf{G} \cdot \mathbf{F} \) that has dimension \( M_I \times (2N_f + 1) \). Since, in general, \( M_I > 2N_f + 1 \), we solve for \( \hat{\mathbf{x}} \) using a pseudoinverse, i.e.,

\[
\hat{\mathbf{x}} = (\mathbf{A}^T \cdot \mathbf{A}^{-1}) \cdot \mathbf{A}^T \cdot \mathbf{W} \cdot \mathbf{v}_{ADC}.
\]

This, then, is an estimate of the spectral amplitudes of both the interferers and the message. The size of \( \mathbf{A}^T \cdot \mathbf{A} \) depends on the number of expansion frequencies assumed and not the number of time samples. In cases where the bandwidth is small, the matrix inverse is easily computed. For a larger bandwidth, an iterative approach may be faster [9].

The error vector magnitude (EVM) is defined as EVM \( \equiv \| \mathbf{x}_m - \mathbf{x}_{mo} \| / \| \mathbf{x}_{mo} \| \) where \( \mathbf{x}_m \) is the subset of \( \mathbf{x} \)
corresponding to the retrieved message when the interferer is present, and $\hat{x}_{m,o}$ is the retrieved message with no interferer.

B. Simulated Results: Known Interferers

Here, two cases are presented where the aforementioned analysis is applied. In both cases, the ideal message and interferers are passed through the ideal nonlinear amplifier model described by (1). The power levels and the input-referred third-order intercept point ($\text{IIP}_3 \equiv 13.3 \times k_1/k_3 \text{ mW}$) were chosen, such that the intermodulation sidebands bury the message signal, making it impossible to receive without the selective sampling technique.

The first case is a very simple one. A small message tone of $-80 \text{ dBm}$ is present at $240 \text{ MHz}$, along with two interferers, each with $-29 \text{ dBm}$ power at $270$ and $300 \text{ MHz}$. The amplifier is modeled by $25 \text{ dB}$ gain and $-9 \text{ dBm}$ $\text{IIP}_3$ [$k_1 = 18$, $k_3 = -1906 \text{ V}^{-2}$ in (1)]. This choice of interferer and nonlinearity ($\text{IIP}_3$) generates a third-order tone on the message frequency at a power level of $11 \text{ dB}$ higher than that of the message. Conventional calculation shows that the $\text{IIP}_3$ of the amplifier would have to be increased by $15 \text{ dB}$ to reduce the intermodulation distortion to a level that produces an EVM less than $10\%$. Such an increase would require a substantial increase in dc power.

Fig. 3 shows that the EVM drops when a smaller fraction of the total sample set is retained. The sampling frequency was $2 \text{ GHz}$, and the total sample set was $N_p = 5000$ ($8 \mu s$). Every eight samples were integrated ($N_I = 8$), and thus, the ADC must convert no faster than $4 \text{ ns/sample}$ ($250 \text{ MHz}$). The aforementioned analysis was applied with only three expansion frequencies: $240$, $270$, and $300 \text{ MHz}$ and dc [see (3)]. The dashed line is the result when the interferers are $90^\circ$ out of phase with each other. Its jagged response is due to the changing distribution of $\{t_n\}$ as the threshold drops. If the phase relation between interferers is changed, the curve changes because the points in time, where the interferer drops below the threshold, also change. The solid line averages the EVM for five interferer phase relations. The results show that the error drops below $10\%$, when less than $45\%$ of the samples are retained for the analysis.

Given a particular interferer, each $N_s/N_p$ fraction can be mapped to a threshold voltage. In Fig. 3, the mapping for the interferer corresponding to the dashed line is shown in the figure axes. For example, when $30\%$ of the samples are taken, the corresponding threshold is $20\%$ of the peak signal voltage.

Fig. 4 shows a second case, which roughly corresponds to the digital television (DTV) white-space problem. A message signal consists of three equal magnitude tones at $293$, $294$, and $295 \text{ MHz}$, with an average power of $-80 \text{ dBm}$. An interferer exists in an adjacent channel at $300 \text{ MHz}$, with a $6\text{-MHz}$ bandwidth and an average power of $-8 \text{ dBm}$. The interferer is an $8\text{VSB}$ modulation at a $10.9\text{-MHz}$ symbol rate for an $8-\mu s$ duration (brick-wall filter and no pilot.) The interferer peak-to-average ratio is $5-6 \text{ dB}$. The amplifier is modeled as having $10-\text{dBm} \text{IIP}_3$. The left inset in Fig. 4 illustrates the input spectrum (log scale), and the right inset illustrates the output spectrum if all samples were retained. The nonlinearity causes shoulders that cover the message signal.

Once again, the plot shows the EVM versus fraction of samples accepted. The sampling frequency is $2 \text{ GHz}$ and the total sample set was $N_p = 16,000$. The integration interval is $8$ samples ($N_I = 8$). The analysis in part A was applied with expansion frequencies of $\{0, 293, 294, 295\} \text{ MHz}$, with a total power of $-80 \text{ dBm}$. The interferer was an $8\text{VSB}$ random sequence at $297-303 \text{ MHz}$, with a total power of $-8 \text{ dBm}$. $\text{IIP}_3 = 10 \text{ dBm}$. $100$ sequences of $T_{\text{dur}} = 8 \mu s$ were averaged. Solid line: no noise. Dashed line: with $-95-\text{dBm}$ noise and $20-\text{MHz}$ bandwidth centered at $300 \text{ MHz}$. Left inset illustrates sample amplifier input spectrum (log amplitude). Right inset shows corresponding output when no samples discarded; distortion masks message.

Fig. 5. EVM of the message versus fraction of samples retained. Message signal was a single-tone $-80-\text{dBm}$ tone at $240 \text{ MHz}$. The interferer was two tones at $(270, 300) \text{ MHz}$ with amplitudes $(0.011, -j \times 0.011)$ (dashed line) corresponding to a total average power of $-26 \text{ dBm}$. The relative phases of the interferers were incremented by $45^\circ$ five times and the resulting six EVMs averaged (solid line). Left inset illustrates sample amplifier input, and right inset illustrates how third-order sidebands mask the message.
drops below 10%, when the smallest 4500 of the overall 16,000 samples are used in the algorithm.

When the number of samples becomes very small, the condition number of $A$ becomes large and the solution becomes sensitive to noise. To investigate the effect of noise, we added a band-limited normally distributed noise signal to the input of our model amplifier. A different realization of the noise was added to each of the 100 realizations of the interferer used in Fig. 4. The EVM results were averaged to produce the dashed line in Fig. 4. With noise, the EVM reaches a minimum as the number of samples is reduced ($V_T$ decreasing) and then increases due to the poorer conditioning. This is the major factor limiting how large an interferer (or how low a $V_T/V_{MAX}$) can be tolerated.

IV. FORMULATION: UNKNOWN INTERFERER SPECTRUM

A. Expansion With Subdomain Functions

In the previous examples, it was assumed that the interferer and message frequencies are known precisely and then used in the expansion (3). Usually, a receiver will not know the exact frequencies of an interferer. The LMS fit in (5) is sensitive to the assumed spectrum. To accommodate the possibility that the expansion frequencies do not match up with the interferer’s spectrum, we suppose a modulation of the complex amplitude of each expansion sinusoid. The modulation is approximated by a series of overlapping triangle functions (a type of linear spline) that span the sample record.

We divide the duration of the sample record $T_{dur}$ into $N_{seg}$ segments of equal length, i.e., $T_{seg} = T_{dur}/N_{seg}$. An expansion triangle is centered at the boundaries between each segment. This leads to the following expansion of the interferer:

$$v_{oE}(t_m) = c_o + \sum_{n=1}^{N_I} \sum_{i=0}^{N_{seg}} \left[ c_{ni} \text{tr}(t_m - iT_{seg}) e^{j\omega_{n_i} t_m} \right]$$

where $\text{tr}(\tau) = 1 - |\tau|/T_{seg}$ for $|\tau| < T_{seg}$ and zero otherwise. Thus, each triangle function covers two segments and overlaps its neighbors. The modulation expansion can be retrieved from $x$ and used to calculate EVM in the examples that follow.

B. Simulated Results: Unknown Interferers

The EVMs of the first two examples are plotted in Fig. 5. In those examples, the interferer consists of two tones with equal magnitude, but different relative phase (see caption). A single-tone message signal is located at a frequency where it is covered by the third-order intermodulation of the interferer tones. We chose expansion frequencies that span 297–303 MHz and plotted results for two different densities. The interferer frequencies are chosen for a worst case scenario, where the unknown interferers happen to be located between the expansion frequencies.

As in the previous results, Fig. 5 shows that the error increases when more distorted samples are included. The EVM curves vary as sample times redistribute when the relative interferer phase changes. The solid lines show results, where each point is an average of five EVMs taken as the interferer relative phase increments by 45°. The dashed line is the one member of that set that corresponds to an interferer relative phase of 45° (useful for verification.)

The better results occur when the expansion frequencies are more densely spaced (40-MHz spacing, $N_{fI} = 15$); only three triangle functions ($N_{seg} = 2$) are necessary for good performance. When a less dense set of expansion frequencies are chosen (80-MHz spacing, $N_{fI} = 8$), the EVM becomes sensitive to the number of segments used. For the less dense set, the curve plotted in Fig. 5 corresponds to $N_{seg} = 6$. Using 8 segments gave similar results while using 7 segments produces much better results.
to noise, and this limits the degree to which the time-selective sampling approach can ameliorate the effects of a nonlinearity. Noise can be generated in the conventional sense, but if the expansion functions do not sufficiently cover the signal space of the interferer, then the uncovered interferer also acts as noise in the LMS fit. Thus, the fitting method described in Section IV is critical in cases where the interferer support is not well known. Further algorithm development should improve conditioning and reduce sensitivity to noise.

The simple cubic model of the LNA used in this brief assumed no memory effect. The measurements described in [3] show no evidence of this effect, but they did not measure very low-level signals. Simulations not included here show that to minimize memory effects the amplifier must have a large bandwidth, but not unreasonably large for current technology.

This brief shows how an RF sampling receiver could be designed to mitigate the effects of amplifier nonlinearity when large interferers are present. The concept eliminates the need for reconfigurable filters prior to low-noise amplification. The algorithm and associate parameters (such as sampling and ADC speed) are reasonable for implementation in current CMOS technology.

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**REFERENCES**


