

Name:

ID:

ECE 314 - Introduction to Probability and Random Processes, Spring 2012

Quiz #3

In class: 02/15/2012

Exercise 1. A box containing 8 red, 3 white, 9 blue balls. If 3 balls are drawn at random without replacement determine the probability

(a) that all 3 are red,

Solution:

Method 1: Let $R_i = \{\text{red ball on } i\text{-th draw, } i = 1, 2, 3\}$.

$$P(R_1 \cap R_2 \cap R_3) = P(R_1)P(R_2|R_1)P(R_3|R_1 \cap R_2) = \binom{8}{20} \binom{7}{19} \binom{6}{18} = \frac{14}{285}$$

Method 2:

$$P(\text{all three balls are red}) = \frac{\# \text{ of selections of three out of eight red balls}}{\# \text{ of selections of three out of twenty balls}} = \frac{\binom{8}{3}}{\binom{20}{3}} = \frac{14}{285}$$

(b) that 2 are red and 1 is white,

Solution:

$$\begin{aligned} P(\text{two are red and one is white}) &= \frac{(\text{select two out of eight red balls})(\text{select one out of three white balls})}{(\text{select three out of twenty balls})} \\ &= \frac{\binom{8}{2} \binom{3}{1}}{\binom{20}{3}} = \frac{7}{95} \end{aligned}$$

(c) that none is white,

Solution:

$$P(\text{none is white}) = \frac{\binom{17}{3}}{\binom{20}{3}} = \frac{34}{57}$$

(d) that 1 of each color is drawn,

Solution:

$$P(\text{one of each color is drawn}) = \frac{\binom{8}{1} \binom{3}{1} \binom{9}{1}}{\binom{20}{3}} = \frac{18}{95}$$

(e) that the balls are drawn in the order red, white, blue.

Solution:

Method 1:

$$P(\text{balls are drawn in order red, white, blue}) = \frac{1}{3!} P(\text{one of each color is drawn}) = \frac{1}{6} \times \frac{18}{95} = \frac{3}{95}$$

Method 2:

$$P(R_1 \cap W_2 \cap B_3) = P(R_1)P(W_2|R_1)P(B_3|R_1 \cap W_2) = \binom{8}{20} \binom{3}{19} \binom{9}{18} = \frac{3}{95}$$

You don't need to evaluate the final numerical values.