ECE 314 - Introduction to Probability and Random Processes, Spring 2013
Homework #8
Due: 04/17/2013 (in lecture)

Exercise 1. *Jointly Gaussian random variables.* Let X and Y be jointly Gaussian random variables with pdf

$$f_{X,Y}(x,y) = \frac{1}{\pi\sqrt{3/4}} e^{-\frac{1}{2}\left(\frac{4}{3}x^2 + \frac{16}{3}y^2 + \frac{8}{3}xy - 8x - 16y + 16\right)}.$$

Find $E(X), E(Y), \text{Var}(X), \text{Var}(Y), \text{Cov}(X,Y)$.

Exercise 2. *One discrete and one continuous random variables* Let the signal S be a random variable defined as follows:

$$S = \begin{cases} -1, & \text{with probability } \frac{1}{3} \\ 0, & \text{with probability } \frac{1}{3} \\ +1, & \text{with probability } \frac{1}{3}. \end{cases}$$

The signal is sent over a channel with additive Laplacian noise Z, i.e., Z is a Laplacian random variable with pdf

$$f_Z(z) = \frac{\lambda}{2} e^{-\lambda|z|}, \quad -\infty < z < \infty.$$

The signal S and the noise Z are independent and the channel output is their sum $Y = S + Z$.

(a) Find $f_{Y|S}(y|s)$ for $s = -1, 0, +1$.

(b) Now, suppose we use the following decoding rule: If $Y < -1/2$, we estimate S by $\hat{S} = -1$, if $-1/2 \le Y \le 1/2$, we estimate S by $\hat{S} = 0$ and if $Y > 1/2$, we estimate S by $\hat{S} = +1$. Find the probability of decoding error, i.e., $P(\hat{S} \neq S)$.

Exercise 3. *Correlation coefficient ρ*

(a) Show that $\rho(aX + b, Y) = \rho(X, Y)$

(b) Consider n independent tosses of a coin with probability of "heads" in each toss equal to p. Let X and Y be the number of heads and of tails, respectively. What is $\rho(X, Y)$?