Exercise 1. Exponential Random Variable is Memoryless. The time $T$ until a new light bulb burns out is an exponential random variable with parameter $\lambda$. Jane turns the light on, leaves the room, and when she returns, $t$ time units later, finds that the bulb is still on, which corresponds to the event $A = \{T > t\}$. Let $X$ be the additional time until the bulb burns out. What is the conditional CDF of $X$, given the event $A$?

Exercise 2. Let $X$ have the normal distribution with mean 0 and variance 1, i.e.,

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$ 

Also, let $Y = g(X)$ where

$$g(t) = \begin{cases} -t, & t \leq 0 \\ \sqrt{t}, & t > 0 \end{cases}$$

Find the probability density function of $Y$. Hint: what is the interval in which $Y$ is nonzero?

Exercise 3. The random variable $X$ has probability density function:

$$f_X(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

The value $X$ is processed by a clipping circuit to yield the output $Y$ as:

$$Y = \begin{cases} \frac{1}{2}, & 0 \leq X \leq 1 \\ X, & X \geq 1. \end{cases}$$

(a) Find $E[X]$, $E[X^2]$ and $E[X|X \geq 1]$.
(b) Find $f_Y(y)$, the probability density function of $Y$.
(c) Find $E[Y]$, $E[Y^2]$ and $E[Y|X \geq 1]$.

Exercise 4. Let the random variable $N(t)$ be the number of packets arriving at a network router during time $(0, t]$. Suppose $N(t)$ is Poisson with pmf

$$p_N(n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

for $n = 0, 1, 2, \ldots$.

(a) Find the CDF of $N(t)$.
(b) Let the random variable $Y$ be the time to get the $n$-th packet. Find the pdf of $Y$. 

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