ECE 314 - Introduction to Probability and Random Processes, Spring 2013
Homework #6
Due: 03/29/2013 (in lecture)

Exercise 1. *Exponential Random Variable is Memoryless.* The time T until a new light bulb burns out is an exponential random variable with parameter λ. Jane turns the light on, leaves the room, and when she returns, t time units later, finds that the bulb is still on, which corresponds to the event $A = \{T > t\}$. Let X be the additional time until the bulb burns out. What is the conditional CDF of X, given the event A?

Exercise 2. Let X have the normal distribution with mean 0 and variance 1, i.e.,

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Also, let $Y = g(X)$ where

$$g(t) = \begin{cases} -t, & t \leq 0 \\ \sqrt{t}, & t > 0 \end{cases}$$

Find the probability density function of Y. *Hint: what is the interval in which Y is nonzero?*

Exercise 3. The random variable X has probability density function:

$$f_X(x) = \begin{cases} \frac{x}{2}, & 0 \leq x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

The value X is processed by a clipping circuit to yield the output Y as:

$$Y = \begin{cases} \frac{1}{2}, & 0 \leq X \leq 1 \\ X, & X \geq 1. \end{cases}$$

(a) Find $\mathrm{E}[X]$, $\mathrm{E}[X^2]$ and $\mathrm{E}[X|X \geq 1]$.

(b) Find $f_Y(y)$, the probability density function of Y.

(c) Find $\mathrm{E}[Y]$, $\mathrm{E}[Y^2]$ and $\mathrm{E}[Y|X \geq 1]$.

Exercise 4. Let the random variable $N(t)$ be the number of packets arriving at a network router during time $(0, t]$. Suppose $N(t)$ is Poisson with pmf

$$p_N(n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

for $n = 0, 1, 2, \ldots$.

(a) Find the CDF of $N(t)$.

(b) Let the random variable Y be the time to get the n-th packet. Find the pdf of Y.