ECE 314 - Introduction to Probability and Random Processes, Spring 2013
Homework #3
Due: 02/20/2013 (in lecture)

Exercise 1. A coin (with probability p of flipping heads and probability $q = 1-p$ of flipping tails) is flipped 15 times. What is the probability that the 15-th flip is either the 10-th head or the 12-th tail.

Exercise 2. A particular class has had a history of low attendance. The annoyed professor decides that she will not lecture unless *at least* k of the n students enrolled in the class are present. Each student will independently show up with probability p_g if the weather is good and with probability p_b if the weather is bad. Given the probability of bad weather on a given day, obtain an expression for the probability that the professor will teach her class on that day.

Exercise 3. We run an experiment and record the discrete random variable X. By running lots of trials, we determine that it has the following probability mass function:

$$p_X(x_k) = \begin{cases} k/2, & x_k = -2 \\ k/4, & x_k = -1 \\ k/2 & x_k = 0 \\ k/4, & x_k = 1 \\ k/2, & x_k = 2 \\ 0, & \text{else} \end{cases}$$

where k is some constant.

(a) Find the value of k that makes this a valid probability mass function.

(b) Find $P(X \geq 0)$.

(c) Find $P(X \leq 1 | X \geq 0)$.

(d) Find $P(X^2 = 4)$.

Now, suppose that we define a new discrete random variable Y by $Y = X^2$. Obviously, we could re-run the experiment and record the discrete random variable Y many times to find the probability mass function, but that would take a lot of time. Thus, we would like to get the probability mass function of Y from that of X.

(e) What is the range of Y? (in other words, what values does Y take with non-zero probability)

(f) Find the probability mass function $p_Y(y_k)$ of Y. (*Hint: Note that this simply means finding the probability that Y takes each of the values in the range you found in (e).*)

Exercise 4. You just rented a large house and the realtor gave you 5 keys, one for each of the 5 doors of the house. Unfortunately, all keys look identical, so to open the front door, you try them at random.

(a) Define the random variable X equal to the number of keys you will need to try to open the door. Find the probability mass function of X under the following assumption: after an unsuccessful trial you mark the corresponding key so that you never try it again.

(b) Repeat part (a) for the case where the realtor gave you an extra duplicate key for each of the 5 doors.

Hint: define K_i as the event that the i-th key selected opens the door.

Exercise 5. Suppose I give a five-point multiple-choice exam, and have figured out that the resulting score X obeys the cumulative distribution function (CDF) given by:

$$F_X(x) = \begin{cases} 0, & x < 0 \\ 1/10, & 0 \leq x < 1 \\ 7/20, & 1 \leq x < 2 \\ 11/20, & 2 \leq x < 3 \\ 13/20, & 3 \leq x < 4 \\ 17/20, & 4 \leq x < 5 \\ 1, & 5 \leq x \end{cases}$$

(a) Find the probability mass function $p_X(x_k)$ for this random variable X.

(b) What is the probability that a student gets a score greater than or equal to 3?

(c) Suppose I define a "passing grade" as getting a score greater than or equal to 3, and I define an "A" grade as getting all 5 correct. What is the probability that a student gets an "A" grade given that they received a "passing grade"?

(d) Suppose that I decide to (possibly) change my grading scale from that in (c). I decide to set the "passing grade" as the largest (integer) score such that at least 60 percent of the people pass. Under this rule, find the score required for a "passing grade".