Exercise 1. Independence. You have 2 five-sided dice. The sides of each are numbered from 1 to 5. The dice are fair, and each die roll is independent of all others.

(a) Event $A$ is you roll both dice, and the total is 10 (i.e., if you add the number that comes up on one die to the number on the other die, the total is 10).

i. Is event $A$ independent of the event at least one of the dice has a 5 showing?

ii. Is event $A$ independent of the event at least one of the dice has a 1 showing?

(b) Event $B$ is you roll both dice and the total is 8.

i. Is event $B$ independent of getting doubles (i.e., both dice are showing the same number)?

ii. Given that you rolled both dice and the total was 8, what is the probability that at least one of the dice has a 3 showing?

iii. Given that you rolled both dice and the total was 8, what is the probability that at least one of the dice has a 5 showing?

Exercise 2. (a) A student has four different pairs of shoes and never wears the same pair on two consecutive days (but he/she always wears a matched pair). In how many ways can he/she wear shoes in 5 days?

(b) A closet contains 8 pairs of shoes. If 4 shoes are selected at random, what is the probability that there is no complete pair? What is the probability that there is exactly one complete pair?

Exercise 3. In Lake Wyola, there are $r$ kinds of fish, and $n$ fish of each kind. When Joe goes fishing, he does not throw the fish back but instead puts them in his cooler. On a summer afternoon, Joe fishes at Lake Wyola and catches 5 fish. (Note: Joe puts them in his cooler because it makes the problem more interesting; we encourage the modern practice of catch-and-release fishing, so that the fish can fight another day - 314 Management.)

(a) What is the probability that the 5 fish he caught are all of the same kind?

(b) What is the probability that the fish are all of different kind? Assume $r \geq 5$.

(c) What is the probability that of the 5 fish, 3 are of one kind and the other 2 are of 2 different kinds?
Exercise 4. Suppose that three trucks leave a warehouse with 20 parts in the first truck, 40 parts in the second truck, and 40 parts in the third truck. Suppose that there are 5, 10, and 2 defective parts out of those in the respective trucks. Suppose I receive my shipment of five parts randomly drawn from those on one of the trucks, where the three trucks are equally likely to have made my delivery.

(a) What is the probability that there are exactly three defective parts out of the five parts in my shipment?

(b) Given that there are exactly three defective parts in my shipment, what is the probability that it came off of the second truck.

Exercise 5. This problem looks like a random variable problem (for those of you who know what that is), but the whole point is that you are able to solve it using what you know already if you can read a graph. This is a very important problem, so focus on solving this by yourself and clearly understanding what is going on. If you do so, ECE 314 life will be easy for quite a while...

Let $X$ be the number of bad chips in a shipment shipped from Xcompany, and let $Y$ be the number of bad chips in a shipment shipped from Ycompany. The reliability sheets from the two companies give the probability of a given number of bad chips in a given order. They display their information graphically, as follows: For example, the probability of getting no bad chips in a shipment from Xcompany is 0.05, the probability of getting one bad chip in a shipment from Xcompany is 0.1, the probability of getting two bad chips in a shipment from Xcompany is 0.15, etc.

(a) Suppose you receive a shipment of chips from Xcompany. Find the probability that the number of bad chips is less than or equal to 3.
(b) Suppose you receive a shipment of chips from Xcompany. Somebody tells you that there are at least two bad chips in the shipment. Given this information, find the probability that the number of bad chips is less than or equal to 3.

(c) Suppose a shipment of chips arrives, but you are not sure from which company it came; however, you know the probability it came from Xcompany is 0.25 and the probability it came from Ycompany is 0.75. Find the probability that the number of bad chips in the shipment is less than or equal to 3.

(d) If you could choose to buy chips from only one company (X or Y), from which company would you buy chips? Be sure to justify your answer.