ECE 314 - Introduction to Probability and Random Processes, Spring 2013
Homework #1
Due: 02/06/2012 (in lecture)

Exercise 1. Note that we key you to the outcome of the experiment by telling you what was recorded - that is the outcome. Write down the sample space for each of the following experiments:

(a) I flip a fair coin three times and record the sequence of flips.

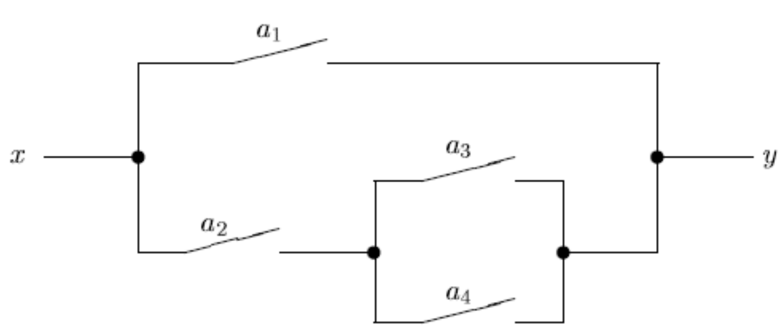
(b) I flip a fair coin three times and record the number of heads.

(c) For lunch, I draw two pieces of fruit (the second *without replacing* the first) from a bag that contains one apple, two bananas, and three oranges, and I record the result without regard to order. (By saying without order, I mean that (apple, banana) is the same outcome - and lunch - as (banana, apple). Each might be recorded as 1 apple and 1 banana).

(d) I shoot an arrow at a circular archery target (assume I always hit the target) of radius 3 and note the distance of the arrow from the center of the target. Also draw (by shading) the sample space as a subset of the real line.

(e) Again I shoot an arrow at a circular archery target (assume I always hit the target) of radius 3, but now I also want to know in which direction I missed. Hence, for each trial, I note polar coordinates of the hit by recording an ordered pair: (distance of the arrow from the center of the target, angle - measured from horizontal to the right - at which the arrow hits). Also draw (by shading) the sample space as a subset of the two-dimensional plane.

Exercise 2. Consider the following communication network between points x and y:



Let A_i denote the event that link a_i is operational (and $\bar{A}_i$ that it is not). Let B be the event that there is communication between x and y.

(a) Express the event B in terms of A_is and $\bar{A}_i$s.

(b) Suppose that $P(A_i) = 0.5$ for all i and the condition that each link is independent of all other links. Find the probability that there is a connection from x to y.

Exercise 3. Consider an experiment with sample space S and events A, B, C and D with the following probabilities: $P(A \cup B) = \frac{5}{8}$, $P(A) = \frac{3}{8}$, $P(C \cap D) = \frac{1}{3}$, $P(C) = \frac{1}{2}$. Furthermore, A and B are mutually exclusive, while C and D are independent.

(a) Find $P(A \cap B)$.

(b) Find $P(B)$.

(c) Find $P(A \cap \bar{B})$.

(d) Find $P(A \cup \bar{B})$.

(e) Are A and B independent?

(f) Find $P(D)$.

(g) Find $P(C \cup D)$.

(h) Find $P(C|D)$.

(i) Find $P(C \cap \bar{D})$.

(j) Find $P(C \cup \bar{D})$.

(k) Find $P(\bar{C} \cap \bar{D})$.

(l) Are C and $\bar{D}$ independent?

Exercise 4. You play a carnival game that consists of throwing three darts at a target. Let D_i be the event that dart i hits the target. Suppose you have the following information:

- D_1 is independent of D_2 and D_3
- $P(D_1) = 0.8$
- $P(D_2) = 0.8$
- $P(D_3|D_2) = 0.9$
- $P(D_3|\bar{D}_2) = 0.5$.

(a) Find $P(D_3)$ the probability that the third dart hits the target.

(b) Given that the third dart hits the target, what is the probability that the second dart hit the target.

(c) Find the probability that all three darts hit the target.

(d) Find the probability that exactly two darts hit the target.

Exercise 5. *De Méré paradox*. Consider the following two games.

Game 1: Roll a fair die 4 times independently and you win if you get at least one 6.

Game 2: Roll two fair dice independently 24 times and you win if you get at least one double sixes.

De Méré argued that the probability of winning is the same for these two games (since the probability of winning in the first game is $4 \times 1/6 = 2/3$ and in the second is $24 \times 1/36 = 2/3$. Is this correct? If not, which game would you rather play and why?