ECE 314 - Introduction to Probability and Random Processes, Spring 2013
Homework #10
Due: 05/01/2013 (in lecture)

Exercise 1. You are attempting to estimate the probability of an event A. Find the number of samples you would have to take to get within ± 0.01 accuracy with 95 % confidence if: (i) $P(A) = 0.5$, (ii) $P(A) = 0.01$.

Exercise 2. You know that a sequence of random variables $X_1, X_2, \ldots$ is independent and identically distributed (i.i.d.), and that each is Gaussian with some *unknown* mean μ and known variance $\sigma^2 = 4$; in other words, $X_i \sim N(\mu, 4)$ for all i. Your boss asks you to estimate the mean μ, so you head to the lab and collect the following observations:

$$\begin{aligned} X_1 &= 3.4 \\ X_2 &= 3.8 \\ X_3 &= 0.0 \\ X_4 &= 2.7 \\ X_5 &= 0.8 \\ X_6 &= 1.2 \\ X_7 &= 0.6 \\ X_8 &= 1.3 \\ X_9 &= 0.7 \\ X_{10} &= 0.7 \end{aligned}$$

(a) Find a (good) unbiased estimate $\hat{\mu}$ of the mean μ.

(b) Find the variance of your estimator from (a).

(c) Find an interval $[a, b]$ such that you are 95% confident that $\mu \in [a, b]$.

(d) Your boss is unhappy with the length of the uncertainty interval you have provided in part (a), and asks you to cut its length in half. How many more observations would you need to accomplish such?

Exercise 3. Let $X_1, X_2, X_3, \ldots$ be independent Gaussian random variables, each with density function $f_X(x)$ with unknown mean $E[X] = \mu$ and variance 1.

Suppose that we observe 100 variables in this sequence and estimate the mean μ as:

$$\hat{\mu} = \frac{1}{100} \sum_{i=1}^{100} X_i = 4.5$$

- Give an interval $[a, b]$ such that $P(\mu \in [a, b]) = 0.99$.
- Find the minimum number of samples N that we must take such that $|\mu - \frac{1}{N} \sum_{i=1}^{N} X_i| < 0.05$ with: (i) 95% confidence, (ii) 99% confidence