ECE 314 - Introduction to Probability and Random Processes, Spring 2013
Homework #5
Due: 03/13/2013 (in lecture)

Exercise 1. *The Poisson Random Variable.* A Poisson random variable has a pmf given by

$$p_X(k) = e^{-\lambda}\frac{\lambda^k}{k!} \quad k = 0, 1, 2, \ldots$$

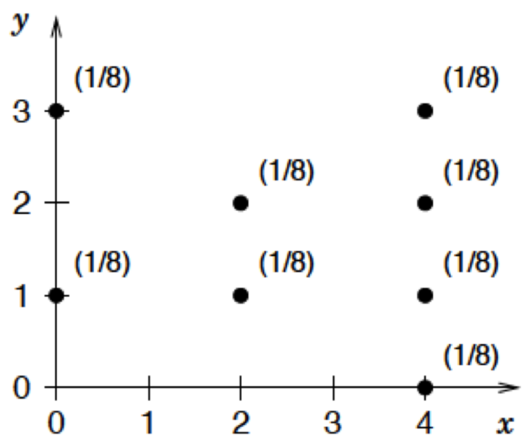
where λ is a positive parameter.

(a) Prove that $p_X(k)$ is a *legitimate* pmf.

(b) Calculate the expected value $E[X]$.

(c) Calculate the variance Var(X).

Exercise 2. Consider an outcome space comprising eight equally likely event points, as shown below: (a)



Which value(s) of x *maximize(s)* $E[Y \mid X = x]$?

(b) Which value(s) of y *maximize(s)* Var$(X \mid Y = y)$?

(c) Let $R = \min(X, Y)$. Prepare a neat, fully labeled sketch of $p_R(r)$.

(d) Let A denote the event $\{X^2 \geq Y\}$. Determine numerical values for the quantities $E[XY]$ and $E[XY|A]$.

Exercise 3. The random variable X has probability density function

$$f_X(x) = ce^{-|x|} \quad \text{for} \quad -\infty < x < \infty$$

(a) Find the value of c.

(b) Find $P[X \leq 0]$.

(c) Find $P[-1 \leq X \leq 2]$.

(d) Find $P[X^2 \geq 4]$.

(e) Find $E[X]$.

(f) Find $E[X^2]$.

(g) Find the variance of X.

(h) Find and sketch the cumulative distribution function $F_X(x)$.

Exercise 4. An absent-minded chemistry professor forgets to label two identically looking bottles. One contains a chemical named Iocane and the other contains a chemical named Sennari. It is well known that the radioactivity level of Iocane has the Unif$[0, 1]$ distribution, while the radioactivity level of Sennari has the Exp(1) distribution.

(a) Let X be the radioactivity level measured from one of the bottles. Let $\Theta = 0$ denote the case in which the content of the bottle is Iocane and let $\theta = 1$ denote the case in which the content of the bottle is Sennari. Also, $P(\Theta = 0) = P(\Theta = 1) = \frac{1}{2}$. The professor decide to label the bottle "Iocane" according to the rule

$$D(x) = \begin{cases} 0, & P(\Theta = 0|X) > P(\Theta = 0|X) \\ 1, & \text{otherwise} \end{cases}$$

Find a final expression for $D(x)$.

(b) What is the associated probability of error?

Exercise 5. You have just finished fixing your grandmas television set at noon on a give day. Let τ be a continuous random variable that is the number of hours until the television breaks again. You know how bad your electrician skills are and thus you are able to figure out that $P[\tau \geq t] = e^{-0.25t}$ for all $t \in [0, \infty)$. In other words, the probability that the television lasts longer than t hours is $e^{-0.25t}$.

(a) Find the cumulative distribution function for τ.

(b) Find the probability density function of τ.

Use your answer from (b) to answer the questions below:

(c) Your grandma wants to watch a show that runs from 2:30 p.m. to 4:30 p.m. that afternoon. Find the probability that grandma is able to watch her entire show.

(d) Find the probability that the television breaks between 2:30 p.m. and 4:30 p.m. that afternoon.

(e) Your grandma calls to tell you that the television is not working at 11:00 p.m. that night. Conditioned on this information, what is the probability that it was still working at 9:00 p.m.?