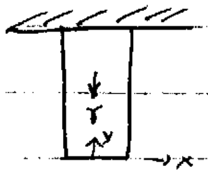
Self weight.



$$\sigma_y = \gamma y$$
$$\sigma_x = \tau_{xy} = 0$$

when a body force is present

$$\sigma_{x,x} + \tau_{xy,y} = 0$$
$$\sigma_{y,y} + \tau_{xy,x} + F_y = 0$$

see Timo + Goodier p. 26

one solution is

$$\sigma_x = \Phi_{,yy} - F_y y \quad \tau_{xy} = -\Phi_{,xy}$$

$$\sigma_y = \Phi_{,xx} - F_y y$$

note $F_y = -\gamma$

so, $\Phi_{,xx} + \gamma y = \gamma y$

$$\Phi_{,xx} = 0$$
$$\Phi_{,x} = f_1(y)$$
$$\Phi = x f_1(y) + f_2(y)$$

$$\sigma_x = x f_1''(y) + f_2''(y) - \gamma y = 0$$
$$\tau_{xy} = -f_1'(y) = 0$$
$$f_1 = c_1$$

$$\Phi = x c_1 + f_2(y)$$
$$\sigma_x = f_2''(y) - \gamma y = 0$$
$$f_2''(y) = \gamma y$$

$$f_2'(y) = \frac{\gamma y^2}{2} + c_2$$

$$f_2(y) = \frac{\gamma y^3}{6} + c_2 y + c_3$$

$$\Phi = x c_1 + \frac{\gamma y^3}{6} + c_2 y + c_3$$

$$\sigma_x = \gamma y - \gamma y = 0 \quad ✓$$

$$\tau_{xy} = 0 \quad ✓$$

$$\sigma_y = +\gamma y \quad ✓$$

since the constants appear on first order terms, $c_1 = c_2 = c_3 = 0$ does not affect the stress.

so $\Phi = \frac{\gamma y^3}{6}$ which also satisfies $\nabla^4 \Phi = 0$
where the RHS is zero since body force independent of x, y

calculate strains

$\varepsilon_y = \frac{\gamma y}{E}$ $\quad \varepsilon_x = -\frac{\nu \gamma y}{E}$ $\quad \gamma_{xy} = 0$

integrate to get u, v

$v = \frac{\gamma y^2}{2E} + f_2(x)$ $\quad u = -\frac{\nu \gamma y x}{E} + f_1(y)$

$\gamma_{xy} = -\frac{\nu \gamma x}{E} + f_1'(y) + f_2'(x) = 0$ finding γ_{xy} will helps us find $f_1(y)$ $f_2(x)$

$$f_1'(y) = 0 \rightarrow f_1(y) = c_1$$

$$f_2'(x) = \frac{\nu \gamma x}{E} \rightarrow f_2(x) = \frac{\nu \gamma x^2}{2E} + c_2$$

at this point we must solve for c_1 and c_2

at $x = 0$ $u = 0$ by symmetry

$$u(x=0) = f_1(y) = c_1 = 0$$

$$\boxed{c_1 = 0}$$

$$\boxed{u = -\frac{\nu \gamma x y}{E}}$$

$v(y=L) = 0$ at the top of the element.

$$v(y=L) = \frac{\gamma L^2}{2E} + \frac{\nu \gamma x^2}{2E} + c_2$$

this cannot be zero for all x unless $\nu = 0$, so we cannot exactly satisfy the displacement boundary condition.

We can try an <u>approximate</u> version of the b.c.

$$v(x=0, y=L) = 0$$

$$\frac{\gamma L^2}{2E} + c_2 = 0$$

$$\boxed{c_2 = \frac{-\gamma L^2}{2E}}$$

$$v = \frac{\gamma}{2E}\left(y^2 - L^2\right) + \frac{\nu \gamma x^2}{2E}$$