University of Massachusetts - Amherst
Department of Civil & Environmental Engineering
CEE 630: Advanced Solid Mechanics (Fall 2007)

Midterm Exam (1 hour 15 minutes): October 23

All problems worth 25 points.

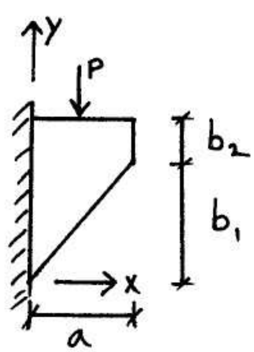
Problem 1: Consider the state of stress defined below:

$$\boldsymbol{\sigma} = \begin{bmatrix} -2 & 0 & -\sqrt{\frac{13}{4}} \\ 0 & 1 & 0 \\ -\sqrt{\frac{13}{4}} & 0 & 4 \end{bmatrix}.$$

(a) Sketch the non-zero stresses on a cubic material element. Make sure to indicate a coordinate system and take care of signs.
(b) Find the principal stresses.
(c) Find one principal direction. If you did not find the principal direction using eigen-analysis, show that the direction you found corresponds to an eigenvector of the stress matrix.
(d) For what value of the Poisson's ratio ν does this state of stress represent the solution to a plane strain problem.

Problem 2: The structural element shown in figure (a) below is a corbel, used to transfer vertical loads from a beam end or arch springing into a column or wall. Figure (b) shows a gross idealization of such a structural element.

(a) 

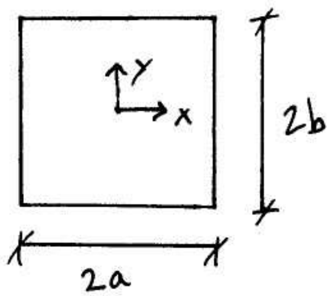
(b) 

(a) Would a plane stress or plane strain model of a corbel be more accurate in your opinion? What would affect this accuracy?
(b) Write down as complete a set of traction boundary conditions as possible for the corbel, including along $x = 0$. State clearly any assumptions you are making, and evaluate whether they are reasonable or not. Your answer may be in terms of the constants a, b_1, b_2.

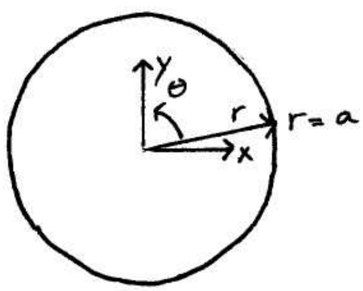
Problem 3: Consider the stress function for 2D elasticity

$$\Phi(x,y) = c_1 x^6 y^2 + c_2 x^3 y^3 + c_3 x^3 y$$

(a) Determine as much as you can about the constants $c_1,\ c_2,\ c_3$ by enforcing compatibility.
(b) Calculate the stresses resulting from the stress function after substitution of results from (a).
(c) Calculate the tractions on the boundary of a plate of dimension $a \times b$ (see figure below), and sketch these tractions on a drawing of the plate.



Problem 4: Consider the circular cross section shown below. Assume that a membrane inflated over a circular area has height of the form $z = c_1 \frac{p}{s}(r^2 - c_2^2)$, where p is the pressure, s is the membrane stress, and $c_1,\ c_2$ are constants.



(a) Determine the constants c_1 and c_2 using boundary conditions and the governing equation $\bigtriangledown^2 z = -\frac{p}{s}$. Note: $\bigtriangledown^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$.
(b) Show using the membrane analogy for torsion that $J = \frac{\pi^2 a^4}{2}$ for a circular cross section.