

University of Massachusetts - Amherst  
Department of Civil & Environmental Engineering  
CEE 630: Advanced Solid Mechanics (Fall 2009)

**Homework #7: Due November 12**

**Problem 1:** Consider a simply supported, prismatic beam of length  $2L$ , moment of inertia  $I$ , elastic modulus  $E$ . A transverse force  $P$  is applied at the midspan, which corresponds to  $x = 0$ . Assume a displacement function of the form

$$v(x) = \sum_{i=1}^n a_i (x^{2i} - L^{2i})$$

with  $x$  defined so that  $x = 0$  at the midspan, and the supports are at  $\pm L$ .

(a) For  $n = 2$  calculate, by hand and using the Rayleigh-Ritz method, the values of  $a_1$  and  $a_2$ . Find the midspan displacement and compare to the exact solution from beam theory. Consider only bending stresses in your calculation.

(b) Calculate the bending moment by taking derivatives of the expression found in (a). Comment on how this moment diagram compares to the actual moment diagram for this structure.

(c) For some  $n \geq 4$  calculate the coefficients  $a_i$  and the midspan displacement. Discuss the relative magnitudes of the coefficients and comment on convergence of this series solution. Note: You may want to use a symbolic math program like MATLAB to do this calculation.

**Problem 2:** Redo part (a) of problem 1 with  $n = 1$  including the contribution due to shear. Comment on your result. You may substitute reasonable numerical values to make the calculation easier if you prefer.

**Problem 3:** 10.32 Compare your result to the exact solution obtained from beam theory. You may look the exact solution up, no need to calculate it.

**Problem 4:** Approximately how many hours did you spend on this assignment?