Example Eigenvalue Problem:

Let $A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$

the eigenvalue problem is given by

$$(\underline{A} - \lambda \underline{I})\, \underline{x} = \underline{0}$$

in which we require

$$\det(\underline{A} - \lambda \underline{I}) = 0$$

$$\begin{vmatrix} 3-\lambda & 2 \\ 3 & -2-\lambda \end{vmatrix} = (3-\lambda)(-2-\lambda) - 6 = 0$$

$$-6 - \lambda + \lambda^2 - 6 = 0$$

$$\lambda^2 - \lambda - 12 = 0$$

$$\lambda = \frac{1 \pm \sqrt{1+48}}{2} = 4, -3$$

find the eigenvector corresponding to $\lambda_1 = 4$

$$\begin{array}{l} (1) \\ (2) \end{array} \begin{bmatrix} 3-(4) & 2 \\ 3 & -2-4 \end{bmatrix} \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(1) $\quad -x_1^{(1)} + 2x_2^{(1)} = 0$

$$x_2^{(1)} = x_1^{(1)} / 2$$

$$x^{(1)} = [2 \quad 1]^T \frac{1}{\sqrt{5}}$$

find eigenvector corresponding to $\lambda_2 = -3$

$$\begin{array}{c} (1) \\ (2) \end{array} \begin{bmatrix} 3+3 & 2 \\ 3 & -2+3 \end{bmatrix} \begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(1) $6\,x_1^{(2)} + 2\,x_2^{(2)} = 0$

$$x_2^{(2)} = -3\,x_1^{(2)}$$

$$x^{(2)} = [1 \quad -3]^T \frac{1}{\sqrt{10}}$$