ECE 673: Homework 2

Due: March 9 (On-Campus Students);
One week after watching Lecture 9 (Off-campus students).

(1) Consider a queueing system in which jobs always arrive in pairs. The pairs arrive according to a Poisson process with rate $\lambda$. Service is FCFS, with service time being exponentially distributed with parameter $\mu$ per job. (That is, while jobs arrive in pairs, they are served individually).

  * (a) Draw the Markov chain for this queue and write the balance equations.
  * (b) Obtain $\Pi(z) = \sum_{i=0}^{\infty} \pi_i z^i$ for this system, assuming steady state exists.
  * (c) Use the generating function obtained in (b) above to find expressions for $\pi_0, \pi_1, \pi_2, \pi_3$. From these expressions, guess the general expression for $\pi_n$ for any $n$. Then, check that your guess is correct by verifying that your expressions for $\pi_n$ satisfy the balance equations.

(2) Prove that the union of independent Poisson processes with rates $\lambda_1$ and $\lambda_2$ is itself a Poisson process with rate $\lambda_1 + \lambda_2$. You can use the definition of Poisson processes provided in Ross (Chapter 2) or in Kleinrock (also Chapter 2).

(3) Consider a queue in which customers arrive according to a Poisson process with rate $\lambda$ and have exponentially distributed service time, with parameter $\mu$. The queue has one server. The customers exhibit impatience: if a customer is in position $i$ of the queue, he leaves without waiting any further, with a probability of $i\gamma\Delta t + o(\Delta t)$ over an interval of time $\Delta t$.

  Draw the Markov chain for this queue and write the balance equations for the steady-state probabilities.

(4) You have a five-processor system. Failure occur as Poisson processes with rate $\lambda$ per processor. You have one repairman, who can work on one processor at a time: it takes an exponentially distributed amount of time, with mean $1/\mu$ to repair one processor. Find the steady-state probability that there will be $n$ functional processors in the system.