ECE 608 - Signal Theory, Spring 2006

Final Exam PRACTICE PROBLEMS

Problems that did not make the cut to get onto the final exam. Thus, they have some flaws (too easy, too hard, not well-covered by the class), but they still should serve as some practice.

1. For each of the parts below, assume the standard absolute value norm on the real line. Answer each part via an ϵ (or ϵ and δ) proof using "first principles" only (i.e. definition of open and closed sets, definition of convergence, and definition of continuity, respectively).

   (a) Consider two sequences of real numbers $\{x_n\}$ and $\{y_n\}$ such that $x_n \to x$ and $y_n \to y$. Suppose that $x_n > y_n, \forall n$. What can we say about the relation of the limits x and y? Be sure to justify your answer.

   (b) Let A be the set $\{1, \frac{1}{2}, \frac{1}{3}, \ldots\}$. Find the interior of A.

   (c) Let A be the set $\{1, \frac{1}{2}, \frac{1}{3}, \ldots\}$. Find the closure of A.

2. Consider the space of all length-n binary sequences; that is, length-n row vectors where each element is either a 0 or a 1. For example, for $n = 3$, the space consists of $[000], [001], [010], [011], [100], [101], [110], [111]$.

   Define the addition ('+') of two vectors as the element-wise modulo-2 addition ($0 + 0 = 1 + 1 = 0$ and $0 + 1 = 1 + 0 = 1$). Multiplication ('$\cdot$') is defined element-wise as well, except that the only two scalars are 0 and 1. Call this signal space X.

   We desire to build as much structure on this signal space as possible: normed linear space, Banach space, inner-product space, Hilbert space. Define the norm $\|x\|$ for $x \in X$ by the number of non-zero elements in x. For example, for $n = 3$, $\|[001]\| = 1$ and $\|[101]\| = 2$.

   For arbitrary n (i.e. make sure your proof works for any n):

   (a) Prove that this is a valid norm.

   (b) Is this a Banach space?

   (c) Find an inner product that induces this norm.